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**Kant's Analytic-Geometric Revolution: Ostensive Judgment as
Algebraic Time–State Relation in the *Critique of Pure Reason***

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by

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Dedication

For my parents, Roger and Jackie Heftler.

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Kant's Analytic-Geometric Revolution: Ostensive Judgment as Algebraic Time–State Relation in the *Critique of Pure Reason*

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In the *Critique of Pure Reason*, Kant defends the reality of the mathematically determined world described by classical physics by arguing that such a world is a necessary consequence of the way in which *sensations are brought to understanding*. Knowing is active—it constructs the unity of nature by combining appearances in certain mandatory ways. What is mandated is that sensible awareness provide objects that conform to the structure of ostensive judgment: “This (S) is P.”

Sensibility alone provides no such objects, so the imagination compensates by combining passing point-data into “pure” referents for the subject-position, predicate-position, and copula. The result is a cognitive encounter with a *generic physical object* whose characteristics—*magnitude*, *substance*, *property*, *quality*, and *causality*—are abstracted as the Kantian categories. Each characteristic is a product of “sensible synthesis” that has been “determined” by a rule that is a “function of unity” contained in the subject-position, predicate-position, or copula.

Understanding the possibility of such determination by judgment is the chief difficulty for any rehabilitative reconstruction of Kant's theory. I will show that Kant conceives of sensible synthesis as an act of *line-drawing*, and of the functions of unity as rules for determining how I am to “attend” to this act. The subject-position constructs

substance, identified as the objective time-continuum, while the predicate-position constructs *quality*, identified as the continuum of state-values constituting the second-order type named by the predicate concept. Both positions thus refer, like algebraic variables, to lines of continuous magnitude, and their relation through the copula is one that determines state-value from time-position, thereby placing all sensations in the objective time order of intersubjective agreement.

Kant's theory of physically constructive grammar is thus equivalent to the analytic-geometric formalism at work in the practice of mathematical physics, which schematizes time and state as lines related by an algebraic formula. *Kant theorizes the subject–predicate relation in ostensive judgment as an algebraic time–state function.* When aimed towards sensibility, “S is P” functions as the algebraic relation $t \rightarrow f(t)$.

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Title Abbreviations for Kant's Writings

- ID* *De Mundi Sensibilis atque Intelligibilis Forma et Principiis*. Translated by William J. Eckoff as *Kant's Inaugural Dissertation of 1770*. New York: Columbia college, 1894.
- JL* *Jäsche Logic*. Translated by Robert S. Hartman and Wolfgang Schwarz as *Logic*. New York: Dover Publications, 1988.
- P* *Prolegomena zu einer jeden künftigen Metaphysik die als Wissenschaft wird auftreten können*. Translated by Paul Carus as *Prolegomena to Any Future Metaphysics That Can Qualify As a Science*. Chicago: Open Court Pub Co, 1986.
- MFNS* *Metaphysical Foundations of Natural Science*. Edited and translated by Michael Friedman. Cambridge, MA: Cambridge University Press, 2004.

Introduction

KANT'S GOALS: DEFENDING MATHEMATICAL PHYSICS AND DEFEATING HUME

In the *Critique of Pure Reason*, Kant defends the objective validity of mathematical physics and defeats Humean skepticism by arguing that a sensible object can only be experienced if it arises, a priori, as a mathematical construction. Kant will argue that there is literally no world for the knower unless it arises as already mathematical in its unity.

For Kant, an experience is an epistemic event. To have an experience of an object is to know something about it, and all knowledge generally is realized through the act of judgment, whose structure is “S is P.” In the case of sensible knowledge, this structure is that of ostensive (sensibility oriented) judgment: “This (S) is P.”

The problem is that sensibility does not present an object structured in a way that can be *true*, i.e., as a *this S* that is *P*. This absence is compensated for by inventing this objective structure in the imagination through the activity of “sensible synthesis.” Sensible synthesis is the way the imagination carries out various combinations of point-moments so as to produce referents for *this S*, *P*, and the copula.

This correspondence between the combination of terms in judgment and the combination of sensations in the object is possible because each grammatical element contains a “function of unity” that acts as a rule of sensible synthesis. These **functions of unity** are what guide my intentionality when I assert “S is P.” To know that “S is P” is true is to know that *P* is an essential component of *S*, one that I realize can be truly combined through the copula. I know the truth of this combination because I have originally made the object by means of it. Moreover, this combination is necessary because the knower is necessarily unitary.

The positive program of the First *Critique* is Kant's attempt to show that these functions of unity combine not only concepts, but also *sensations*, into objects of a priori knowledge. From these necessary combinations Kant will derive his "synthetic a priori" principles of all sensible objects. We find that all of Kant's principles are characterizations of magnitude:

- All objects are extensive *magnitudes* across space and through time.
- Every object rests on a permanent substrate, or *substance*.
- Every sense content is a momentary state, or *property*, of the object.
- Every property is understood to be a state-value in a continuum that represents a higher-order *quality*.
- Every state-value is determined by its position in the time-continuum according to a rule.

How are we to understand the claim that the construction of the mathematico-physical object is directed by rules that are originally functions of unity in judgment? How can rules of discursive combination serve as rules of sensible synthesis? In my dissertation, I will show that Kant's theory of schematism entails that the unity of the sensible object itself be one that is a relation between *magnitudes*. The subject-position is the unity of time-magnitude (substance), and the predicate-position is the unity of continuum of state-values (quality). Their truth-relation is the relation that determines state-value as an *algebraic* function of time.

Sensation arrives a priori embedded in the separation-sustaining frameworks of space and time. I know this manifold, however, as a plurality-in-unity. Since this unity is originally lacking, I must produce it myself through spontaneous acts of sensible

synthesis, which construct the unities that must exist in order for ostensive judgment (judgment aiming towards sensibility) to be possible. What unifies the act of apprehending a way-of-plurality into a single “thought” is the procedural **rule** that governs its construction. For Kant, the rule that governs the creation of images of a certain *kind* is what provides the meaning of a *concept*. The meaning of a predicate is the rule that I use to produce an instance of the kind *P*. To recognize that an instance falls under a concept is to know that the rule named by *P* lets me produce an identical instance as an image.

Most concepts (rules) apply only contingently. A sensible object is truly *red*, for example, only if it happens to be red. But some concepts apply necessarily. This is the case for the concepts that are the rules, if any, necessary for making *any* object that can be known—i.e., as an *S* that is *P*. Kant says that the rules that apply necessarily to any object are the ones that construct it as an *S* that is *P*, and these are the functions of unity at work when I intend a subject, predicate, and their relation of truth through the copula. Kant describes the rules in ostensive judgment as follows:

- The subject-position (*this S*) contains rules that guide the production of space, time, magnitude, body, and substance.
- The predicate-position (*P*) contains rules that guide the production of property and quality.
- The copula (*is*) is the relation of sensible-objective truth, which says that any objective state is what it is by virtue of its position in the **objective time order**, a law-necessitated sequence of states that are the same for everyone.

Kant identifies the rules of sensible synthesis with *grammatical positions*, and says that these rules are the sensible *functions of unity* at work when I intend these grammatical positions. How can this theory be made intelligible?

To answer, we must first know what **sensible synthesis** means. Then we must consider what it would mean for sensible synthesis to “fall under” **grammatical positions**. Ruled acts of sensible synthesis are being *related* in judgment through the copula of truth. How do the elements of judgment function as rules for constructing sensations into the unity of the mathematico-physical object? This is the chief puzzle in Kant’s theory.

Extensive synthesis

I will argue that for Kant the act of sensible synthesis is **schematized** (carried out consciously so as to produce an image) through acts of **line-drawing**. Line-drawing is how I consciously emulate all spontaneous (unconsciously enacted) acts of sensible synthesis, which Kant calls acts of *apprehension under a rule*. Consequently, every imaginary product of synthesis is, at least spatially, a line—and also a magnitude. What individuates each rule is not the resulting line, which they all share, but what is “attended” to while the act is carried out. Space tells me to attend to the external relations of sensible elements (point-moments) that are being synoptically presented. Time tells me to attending to the successive nature of the act of drawing, which emulates the passive occurrence of the passing-away of sensations. Body tells me to see reality as extending across space, as a *real* extensive magnitude. Substance tells to attend to the unity of the agency (mine) behind the act of drawing, which has the effect of positing the sensibility-stimulating force of noumenal reality as a reality in temporal extension.

Intensive synthesis

Reality is schematized by positing, and my necessary unity is what makes the unity of the resulting synthesis necessary—it is the product of an identical agent. That is what I intend to mean when I consider the object to be real—its content is forced on me. The reality of sensation is its givenness as an empirical content that I cannot imagine away. Reality can only be meaningful if I can make it, and this I do through the power of imagination. Imagination is, like noumenal reality, a power that makes sensible contents with conscious intent. Kant bases meaning (knowing that something “is P”) on making. I know what *kind* something is when I know the rule that lets me posit, in my imagination, weakly, a real particular that is identical to that something. To know is to know how to make.

I make the unity of sensible synthesis by drawing a line. Space, time, body, and substance are all instances (the first pair is empty, the second is real) of external relations, or “forms,” by which sensations are a priori separated. But the content of sensation, which indicates its reality, is also, Kant says, a magnitude—i.e., the magnitude of the *force* of reality. Reality is also, a priori, an *intensive magnitude*. Every quality is a range of sensible particulars. A particular instance of *red*, for example, is a range within the higher-order *hue* that can be measured. Orange is *more different* from *red* than *red-orange*. Quality is a priori a continuum of state-values. This is the rule of sensible synthesis that Kant assigns to the predicate-position in ostensive judgment.

A schema relates instance and kind

The pure concept is a single rule that determines a whole dimension of difference—space, time, substance, and quality. What allows the rule to infuse meaning into the resulting combination is the a priori separation between sensible elements. Their *lack* of connection is what provides space for my intervention. But every act of plurality-

combining involves time—elements are taken-up one *after* another. Time is the general framework for *any* combination, and the rule must be a rule of putting-together-over-time. This is how Kant explains the possibility of the interface between concept (rule) and object (rule-combined plurality). The interface itself is called a **schema**. Kant says that time itself only be schematized through line-drawing. For this reason, I conclude, *every other* act of sensible synthesis is schematized through line-drawing as well.

Magnitude: the basis of non-analytic truth-relations

Because the subject- and predicate-positions contain rules that I schematize through line-drawing, these positions themselves consequently refer to magnitudes. This means that these positions actually function, at least in part, as **algebraic variables**—terms that range over continuous magnitudes. This, I will show, is what will allow the subject and predicate to relate to one another in the truth relation that Kant calls “synthetic a priori.” What allows the copula to be a determining relation is the fact that the relata are both *magnitudes*. This is the homogeneity that makes their relation through the copula possible and what provides for the *determination* of the predicate by the subject, which relation is also at work when I assert “S is P” an analytic judgment. What determines the predicate in this case is its containment in the object (referent of the subject). But when I assert “This (S) is P” in ostensive judgment, and I relate time-magnitude to state-magnitude, what is the nature of their truth relation?

A priori unity: unity that precedes the particular

An a priori cognition is one that ascertains something about objects “before they are given to us” [Bxvi]. The only facts that we can “cognize a priori about things is what we ourselves put into them” [Bxviii]. I can only know a priori facts about an object if I have made that object myself. It is only by making objects, by having insight into the

conditions prior to their presentation, that I can know truth “a priori,” i.e., know truth independently of anything outside my own resources of making known objects. When I make an object that essentially contains *P* in its presentation, then I know a priori that it is *P*. Necessary objective truths are possible because I know what is essential to the object and what is not.

For instance, when I posit a red triangle and say that it is *red*, I know that “This (red triangle) is red” is a necessary truth. When I say that *All red triangles are red*, I know that this is true as well. How? Because the logical combination *that I provide* before I produce the instance, the combination of *triangle* and *red*, is prior to any possible self-produced image of a red triangle, and thus *red* applies a priori. The basis of necessary connection in the object is the priority of the unity (if any) that precedes the advent of the object’s realization as a particular or image. Particularity is an essential attribute of the object—it must be a particular, what Kant calls an *intuition*. A given particular is a sensation; a made particular, an image.

Copula as determination of predicate by subject

In analytic truth, the subject is the *condition* of predication, the condition of truth. The subject comes first, and then truth is tested against it. When I assert that “S is P,” I mean that *P* is contained in *S* by a prior act of synthesis, i.e., that of logical combination. Analytic truth is necessary because it relates the predicate to the subject through the *same* relation of logical combination that preceded my act of schematizing the object, which is the referent of the subject-position. The subject (and the schematized object) just *is* a logical combination of predicates. If I assert this relation, by articulating the predicate as a separate utterance is a sequence “S is P” and then intending its recombination in the copula, I am telling a truth.

The only examples of synthetic truth that Kant gives are “propositions” of mathematics. I hold that it is crucially significant that he allows the mathematical *equality operator* ($=$) to function as the copula in one of his examples, “the proposition $7 + 5 = 12$ ” [A164/B205]. It shows that the archetypal relation of synthetic truth is the relation between magnitudes.

It is the thesis of my dissertation that the copula in ostensive judgment is functionally identical to the algebraic domain–range operator (\rightarrow). Since the subject-position refers to time itself as a substrate (spatialized magnitude), and since the predicate-position refers to quality as a continuum of intensive magnitude (also schematized as a line), the relation of the copula must be one about the object’s history, which is a sequence of state-values in time such that each value is *determined* by its time-position. The copula is a law, ultimately discoverable as an algebraic function, whereby state-value is determined by time-value, or $t \rightarrow f(t)$. While ostensive judgment and non-ostensive judgment share the same two-pronged structure, and their relation as condition–conditioned, the nature of this relation (and the possibility of necessary truth) is not logical combination but the algebraic relation of domain and range.

The relation of time and quality through the copula in ostensive judgment is the basis of Kantian causality. Unfortunately, within the *Critique*, causality is coupled with the hypothetical judgment form for architectonic reasons. Outside the *Critique*, however, Kant identifies the rule of causality with the copula. Locating causality in the copula also lets us avoid the problem arising from the fact that compound judgments are not necessary for cognition; only atomic judgment is truly necessary. Since *causality* is a category, the causal relation *must* be in operation when I assert a single atomic judgment. By locating causality in the copula, my theory avoids this problem.

$t \rightarrow f(t)$: the basis of Kant's mathematico-physical principles

Finally, I will show that my theory is vindicated by the fact that the hidden structure of ostensive judgment, " $t \rightarrow f(t)$," when analyzed, contains precisely those principles listed by Kant in the Systematic Presentation. These are all a priori judgments because they all predicate something essential about a consciously performed act of line-drawing:

- All objects are extensive magnitudes because space and time themselves arise as magnitudes through the act of line-drawing that produces spatialized time as the very "image of magnitude."
- Every object rests on a permanent substrate because when I draw a line I am a reality-positing force that is self-identical. This identity is what allows me to gather plurality into an extensive unity. Permanence is schematized as the increase in magnitude. Passing time is accumulated and becomes a growing magnitude. Non-change relates to change through the *cumulative* activity of apprehension.
- Every content that I can abstract is momentary because it is mutable.
- Every quality is a degree of magnitude because only in this way can something about time be known a priori. This is the postulate that makes mathematical physics possible. When I quantify quality, I *understand* quality now as being potentially under the control of a function that determines this value from another value. State-value can be *calculated* because time is a magnitude.
- Finally, every state-value is determined by its position in the time-continuum according to a rule because I must be able to distinguish between objective and

subjective time-orders. Causal law is what determines the series of state-values as a function of time.

So we see that all of Kant's principles of physical objectivity are necessary because they predicate features essential to some way of interpreting the basic act of sensible synthesis—line-drawing. The sensible object is a lawful nexus of magnitudes, parsed into time and quality, and related by some function taking time as independent variable and state as the value of a dependent variable, which thinks the unity of the quality of which that value at time t is an instance. These facts can all be expressed in the accusative as objective principles, and these are precisely Kant's principles.

ADVANTAGES OF THIS INTERPRETATION

The structure of the transcendental object

In my dissertation I thus present, for the first time, a strictly **mathematical interpretation of synthesis** itself, and an **algebraic interpretation of Kant's synthetic a priori judgment**, which must be a ruled relation of magnitudes, time and state. I can only know something, and facts about it a priori, if I make (schematize) it. Sensible synthesis, Kant says over and over, is accomplished through line-drawing, which renders every product of synthesis as a potential magnitude. This is what defeats Hume—the subject and predicate *can* relate a priori yet non-analytically through a function that relates two magnitudes as condition and conditioned. And this is what makes nature a priori calculable by means of mathematical formulas—the object itself is a nexus of magnitudes.

Rendering the generic sensible object as a lawful nexus of magnitudes, combined via " $t \rightarrow f(t)$," also lets us **clarify the nature of the transcendental object** by making

the unity of the object a viable basis for synthetic a priori truth. Each of the categories, necessary concepts of any sensible object, refers to a unity having the schematized form of a line of magnitude. These lines of magnitude can be interrelated a priori in various ways—e.g., they can be compared in space or as numbers. This, in fact, is what allows ordinary (non-dynamic) applied mathematics to have objective reality. Things *arise* as countable, and as measurable both in themselves and across space. Things *arise* in Euclidean space and these relations are known a priori. In these judgments, the subject contains a rule of magnitude ($7 + 5$), or a shape (*triangle*), or some intuitable feature of these magnitude (*straight*). The predicate contains any of these as well—e.g., “ $3 + 4$,” *internal angles 180°*, *shortest*.

But the physical object is not a relation between arbitrary magnitudes, but magnitudes that have been constructed in order to make an object for “This (S) is P,” which aims towards the passing plurality of point-moments. This object is not, then, an arbitrary combination as in the above cases of mathematical judgment. Rather it is something specific—the relation of time-order and quality-continuum. This is the internal structure of the sensible object: something that, to be anything for me, must be an object, in imagination (which contains the particular), having the structure of a fact. The structure of the transcendental object is “This (S) is P,” which is possible because the object is a relation of magnitudes—of substance to quality. The schema of “This (S) is P,” which is the schema of the mathematico-physical object, is “ $t \rightarrow f(t)$.”

I cannot make a unified space or time without making it as a magnitude. That is why the world is mathematical a priori—not only in static applied mathematics, but in dynamic applied mathematics. Time is magnitude—in fact, it is the original (schematized) magnitude. This allows change to be brought into the realm of *a priori calculation*.

The nature of necessary objective unity

My theory also lets us understand how sensible unity can be schematized, and how the **necessary identity of the knower** can enter into the physical object as the necessity of intersubjective agreement, Kant's solution to objective truth under representationalism. The basis of objective unity in the identity of its producing agent also explains the otherwise vague distinction between “**forms** of intuition” and “**formal** intuitions.” The *forms* of intuition are the ways-of-separation that I overcome in the act of line-drawing. I must be able to verify my presence at a point-moment by positing an imaginary datum, and the only way I can bring different positions into unity is by *moving* this point, rather than (say) positing *another* one. External relations in intuition are overcome by motion. This makes the *formal intuitions* of space and time. It also makes the imaginary instances for *body* and *substance* into necessary unities, when I attend to the necessary identity of the force (“I think”) behind the moving point. This is how *reality* becomes a continuous extension across space and through time, i.e., as a body and substance. The unity of the quality is also due to me—I span the continuum of values as one agent of line-drawing, so that the quality-continuum coheres as a unitary concept. This is the mathematical basis for the unity of a universal.

Kant thus holds that the knower is aware a priori of **three media** or ways of separation that it overcomes and combines into the necessary unity of my awareness—**space, time, and quality**. Space and time are the ways of plurality intrinsic to intuition as such. No sensible datum can appear, or be an intuition for me, unless it is already contained within the relational frameworks of space and time. Space is the medium that contains and separates a simultaneous plurality, and time is the medium that contains and separates the plurality of passing. Both of these media fall under “extensive magnitude.” That is to say, their elements are taken as externally related to each other *in intuition*

itself. And so Kant says that these media are forms *of* intuition—they belong to intuition (and are independent of understanding: my power to think concepts and follow rules of image-making). Quality is another a priori dimension of variability. Each content of intuition can vary within the dimension of quality by being more or less some particular quality, which is thus taken a priori as a value on a continuum.

The nature of necessary truth

A priori knowledge entails insight into the internal structure of the object of truth. This is only possible if I make the object—I know that “S is P” by looking within myself. I make an S, but I cannot help doing so intentionally. Before I produce an image falling under S, I must make the concept of S.

Analytic truth vs. synthetic truth

There are two bases of necessary truth for Kant—analytic truth and non-analytic, or “synthetic,” truth. Analytic truth is dealt with by **general logic**; synthetic truth, by **transcendental logic**. In analytic truth, the prior unity is logical combination. In synthetic truth, the prior unity is the irreversible and determinate order of state-values. This order is the order of time itself as substrate, which substrate is the referent of the subject-position, produced by following the rule of line-drawing: *think reality as perduring continuum, or “substance.”* What is being ordered is the value of some quality, the continuum of state-values which is the referent of the predicate-position, produced by the rule of line-drawing: *think reality-content as continuum of “qualitative” difference.* Both unities are constructed by an identical agent, and so are necessary unities. Their relation is the copula is the final unity of the object as a sensible fact, a relation of *this* and *P*.

In non-ostensive judgment, I lay the groundwork for analytic truth through a prior act of logical combination. This produces the a priori internal structure of the object, and thus the a priori internal structure that I will assert through the copula, which will combine *P* with *S* in the same way that it was combined with the other concepts in *S* when I constructed *S* prior to my act of schematism.

The same must be said of the unity of the generic physical object, which is the object of a priori synthetic truth in ostensive judgment. Before the object can be a “This (*S*) is *P*” for me, I must combine two sensible functions of unity into the final unity of the objective time order. This unity is the basis of necessary truths—truths about every *this* that can be *P*. Every *this* is a body, a substance, and instantiates a quantified quality *under causal law*. The order that *is* must be a sequence of state-values in a determinate sequence. This is the objective time order, and the lawful determination is the unity of the copula. This ensures that the existential history of the object is causally determined, and serves as the schema of the inner structure of the transcendental object—sensible truth as the relation between time-position and state-value.

Representationalism and identity theory

The only realities that the Kantian knower can access are its own sensible states. These arrive a priori pluralized as point-moments. It is a condition of sensation that this pluralization is spatial and temporal. These are shared media or “forms” of intuition—ways of separation that all point data share *a priori*.

Since sensations arrive as a plurality of point-moments, their connections can only be presented after they are received, i.e., by me. This is done automatically. But it is also done necessarily, because I am by necessity a unitary knower, and I think the object as a fact, unified as “*S* is *P*.” The original lack of unity produced by the a priori forms of

separation is what make knowably necessary unity possible. If I must make a unity to know and experience the object, then this unity is “really” a priori—because the reality of the object is *nothing* but sensations, and these *must* be unified.

The original lack of objective unity is what how Kant utilizes the *identity theory of truth*.

The identity theory of truth under representationalism

Kant conceives necessary truth about an object as a necessary condition of its being that object. The object is a plurality of presentations in a certain unity—the unity necessary for its being an object having the structure of a fact. To know an objective truth a priori is to know something about all possible objects of kind *F* prior to their being given, and to know this by relying only on the resources internal to my own act of knowing. A priori truths must be truths about how I must make objects in my active mode as a being capable of asserting truth by representation. The truths are *about* these representations. Since the object is “nothing for me” until I make it, I make the object in a ways that must realize “S is P.” The object *itself* is an *S* that is *P*. Since there is no object until I make it, then if I make it as an *S* that is *P*, “S is P” is *really* true.

Kant presupposes the **identity theory of truth**. The identity theory says that the object of “S is P” is actually an *S* that is *P*. The only knowable object is just the object that I myself create in my imagination. I can only *know* anything about the object’s unity if I can produce it. How can my activity enter into the existence of the known object?

As we have seen, Kant holds that real (sensible) objects arrive as *originally* pluralities because itself is constituted to produce sensations in spatial and temporal separation. This means that if there is unity in the sensible object, not only *can* I be the maker of its unity, I *must have been* its maker. This is the prior, ontological unity of the

object itself. I know it because I produced it, previously, spontaneously and unconsciously.

Object-making: a staple of general logic

Object-making seems like a radical thesis, but it is already familiar as the very presupposition that gives general logic objective reality. General logic is the study of **non-ostensive judgment**, i.e., of judgment that refers not to objects given in sensibility but to objects that I make from my store of concepts, which Kant defines as rules of image-making. The objects of general logic are always logical constructions—I make the object as a logical combination of images, in my imagination. I know these images because I have produced them from rules that I can name, such as *red* and *triangle*. I make an image of a red triangle, and this is an instance of the object of my judgment. But the actual object here is a *type* of object. The referent of the subject-position is actually the logical combination of concepts that I have combined prior to making the image: *red* and *triangle*. This unity is valid for all instances of the type *red triangle* because all possible red triangles are already contained in the composite rule *red triangle*. The unity of logical combination is what I then (re-)assert through the copula, “is.” A red triangle *is* red because *red* is a rule that is a *condition for the presentation* of any red triangle. This justifies my assertion that all red triangles *must be* red. The copula of *All red triangles are red* thus represents, as a willful act of assertion, something that is a priori the case about the object, i.e., that its generating rules have already been combined logically. The copula of truth expresses the connection of the predicate in the logical combination that precedes the objective image.

Logical combination and logical subordination

What makes the unity of concepts (potential predicates) in the logical object *necessary*? For Kant, the unity that is supremely necessary is the unity of the knower in knowing the object, which is also the unity of judgment since the unity of the object is expressed as the copula. Each concept that constitutes the object is separate. This separation is expressed in judgment as the separation of subject and predicate. Kant says that the “I”s that think the subject and predicate concepts are “different consciousnesses.” These *must* be combined. The combination of rules is logical combination, so the unity of the self here is logical unity. Kant calls this the analytic unity of apperception. The unity of the object, and the unity of the **subject** concept, is the logical unity of logically combined rules of image-making. This unity is necessary because the “I” that thinks this logical combination is unitary. If the logical unity of the object were nullified, the knower itself would lack unity. **Logical combination** is the unity I think in the subject-position.

When combined, a potential **predicate** loses generality. Each potential predicate has higher generality than the complex in the subject-position. There are more red things than there are red triangles. When I intend a concept in the predicate-position, I think the entire range of complexes to which *red* belongs in logical combination. The unity of **logical subordination** is the unity of consciousness that I think in the predicate-position. My consciousness thinks many red things under *red*. Their togetherness is therefore another necessary unity in the knower itself. The unity of the predicate is just the unity of the rule that produces images.

Parallel functions in different domains

For Kant judgment is the source of all rules of combination. Non-ostensive and ostensive judgment differ by operating on different domains—concepts and sensations, respectively. But they share the same basic structure and same basic relation. The

structure is a two-pronged relation between a subject-position (object) and a predicate-position. The relation is one where the object thought in the subject-position is the condition of the truth of the predicate. The nature of a priori truth becomes specified only when the basic structure and truth-relation is applied to a specific domain. In either case, however, the resulting combination is necessary because epistemic consciousness is unitary—one “I think” knows the object (and the fact) as a plurality-in-unity. The object, in Kant’s words, must be “brought” to the necessary unity of consciousness. What differs are the ways-of-separation being overcome by the acts of combination.

In **non-ostensive judgment**, a priori truth is a relation between (*S*) a logical combination of concepts and (*P*) a concept that subsumes this combination via logical subordination. The object is a logical combination of many rule-beholding consciousnesses. The “I”s that think the subject and predicate concepts are themselves “different consciousnesses.” But any object of knowledge is a unity for a unitary knower. In non-ostensive judgment, the unity of the object is logical combination. The unity of the predicate is the unity of logical subordination. The concept *red* subsumes many red objects, among them is *red triangle*. One knower thinks many red species under *red*, and so this unity is also a necessary one. Finally, predication is true in non-ostensive judgment when the relation of the copula is again logical combination (and *P* is an element of the complex preceding the presentation of the object in imagination).

In **ostensive judgment**, the object is the extensive combination into magnitude of many point-moments, or elementary sense-consciousnesses. Epistemic consciousness is one, and so these combinations are necessary. The unity of the object in space and time is the unity of the subject-position. I can then make a mathematical judgment when I predicate some feature of this extension, as in “This ball is five inches in diameter.” But I can also predicate facts about the object’s existence, which is dynamical. Then I relate the

predicate to the time-continuum. This, as we have noted, is only possible if the predicate refers to a magnitude. So Kant defines *quality* as a continuum of state-values, one of which is instantiated at any moment. Finally, predication is true in ostensive judgment when the relation of the copula is one that determines state-value from time-position.

Thus we see a parallel between the ostensive and non-ostensive judgment. The subject is a combination of elements “in” the object. These are logical combination and extensive magnitude, respectively. The predicate-position is a combination of instances under a kind. In non-ostensive judgment, this is the logical togetherness of species under a kind. In ostensive judgment, this is the togetherness of instances within a continuous magnitude—properties that differ by degree and up to a limit.

Making the sensible object as a nexus of magnitudes

Before there is unity in the object, there is a plurality. I select concepts and combine them prior to making the object in the imagination—as a particular *image*. The prior plurality is what lets me be the source of the truth relation, which expressed an internal relation of an object that is originally a collection of *Ps* related to each other by the relation of the copula. This puts truth directly into the object, as its *formation*.

Sensible elements are also pluralities that are combined a priori as the formation of an imaginary object. The combination of subject and predicate *in this case* cannot be one of logical combination, or the relation of subordination that thinks one rule as a logical combination with another. In this case the combination must be something different. What is it? Kant says that space, time, magnitude, body, substance, quality, property, causality, and change are all schematized through line-drawing. The unities here are not logical combinations, but relations of external or qualitative proximity. Point are *tied together* (linked by the synthesis of “reproduction”) by moving a point, which

carries an identical subject, the agent that posits a point meant as the structural simple of reality, *across* and *through* space and time. This, however, always produces a magnitude.

The only necessary non-analytic relations are mathematical ones

It should be no surprise, then, that Kant only gives mathematical examples when he wants to show synthetic a priori truth as something familiar and uncontested. We are already familiar with the holy grail of Kant's system, the synthetic a priori judgment. These are equations and theorems of arithmetic and geometry. Suddenly, Kant lets us know what the synthetic a priori copula is supposed to be—a relation between magnitudes, or characteristics of magnitudes. For example, as we have seen, the equality operator can serve as the copula of a synthetic a priori truth.

Kant sees mathematical physics as the other realm of synthetic a priori truth. The difference is change, the hallmark of existence. The categories here are—unlike space, body, and magnitude—*dynamical* rather than ostensibly *mathematical*. The mathematical categories are ones whose *sense* does not involve time (although the *construction* of any sensible synthesis does). In the mathematical physics of change, what is being combined are sense contents *over time*. Quality is a value in time, a varying property. Substance is time itself. Substance and property are dynamical categories, whose sense depends on time.

Time is the referent of the subject-position, and this is a magnitude. Quality is the referent of the predicate-position, another magnitude. The third dynamical category, that of causality, is what relates them—state-value is caused by time-position, under a law. The relation in physical synthetic a priori judgment is, like that of strictly mathematical judgment, a relation between magnitudes. But in this case it is specifically a relation

between algebraic variables, where one (the predicate) is determined by the other (the subject). This is Kant's conception of synthetic a priori judgment of the physical type.

Three kinds of a priori judgment

So we see that there are at least three kinds of a priori relation—three ways of combination prior to an imaginary object that provide both the form of the object and the basis of truth via “S is P.” The first is **logical combination** and unity. Before I make the object (image), I make the rule as a logical combination of rules. This is the intended referent of the subject-position: $(F \ \& \ G)$. Asserting “is F ” is an a priori truth. The predicate is part of the subject, and the predicate also subsumes the subject as a species, by virtue of the prior specifying act of combining it with G .

The second kind of a priori relation is mathematical combination and unity. In mathematical judgment, such as “ $7 + 5 = 12$,” the copula is equality or some other relation between arbitrary magnitudes *or* aspects of magnitude (such as *straight* and *shortest*).

The third kind of a priori relation is what I must think in order to assert “This (S) is P” towards the passing plurality of point-data that is my connection with reality. This changing array of point-data, if it presents an object of knowledge, presents a *this* that can be P . The *this* is the condition of P —and both, Kant says, are magnitudes. What determines this relation is a law—a rule for translating one magnitude into another. This can only be a relation between two algebraic variables—one ranging over the time-continuum, the other over the quality-continuum. The copula of objective truth that relates them is the algebraic formula that takes time as an argument and produces (a priori) quality as value. The copula, then, is the domain–range operator “ \rightarrow ”. The

structure of the a priori relation of truth (and the internal structure and unity of the sensible object) is “ $t \rightarrow f(t)$.”

Kant fulfills his promise: nature is a priori mathematical because it is constructed that way by the structure of judgment

It is no wonder that we find Kant’s principles to be one and all mathematical. After all, Kant introduces his project as one that will establish the apriority of mathematico-physical knowledge in a way as strong as the one supporting the apriority of change-irrelevant applied mathematics. Kant’s solution is to make change itself a mathematical determination. This is precisely the essential presupposition of mathematical physics. This presupposition is itself a system of necessary relations, named by concepts, and expressed as necessary truths—Kant’s synthetic a priori principles of all physical objects. Mathematical physics is valid a priori because the world arises through the mathematical construction of line-drawing—not only across space and through time, but in terms of quality, and in terms of change (or causality). And it arises this way as the consequence of a unity knower knowing sensible truth by means of “This (S) is P,” wherein the unity of the *this* and the unity of the *P* are continuous magnitudes, and their combination is an algebraic one, where the continuum of time-values thought under the *this* are taken as determining the continuum of state-values thought under the *P*. Thus not only space and time, but also quality and its change, are determined mathematically *because of the structure of judgment in a necessarily unitary understanding*. The world is a priori mathematical in terms of space, time, quality, and change because ostensive judgment has the structure of an algebraic relation.

THE CENTRALITY OF LINE-DRAWING TO KANT'S EPISTEMOLOGY

Line-drawing is the fundamental act of sensible synthesis

Throughout the *Critique of Pure Reason*, Immanuel Kant repeatedly stresses the importance of the imaginary act of **drawing a line**. Line-drawing, Kant says, is the procedure I must follow in order to “present time” as a unifying container of all my sensible states. And since (under Kant’s assumption of representationalism) sensations comprise the matter of the physical world, line-drawing must therefore be a condition for cognizing the systematic unity of the physical universe. Line-drawing, which produces the very sensible significance of *time*, the all-embracing container of knower and known, is clearly an essential aspect, if not *the* essential aspect, of Kant’s theory of physical constructivism.

Pure concepts as rules of line-drawing

First thesis: line-drawing schematizes synthesis

The synthesis of inner sense into unity by means of line-drawing is the all-embracing and fundamental act of synthesis upon which all other special acts of synthesis depend. My first thesis is that line-drawing is the *general act of sensible synthesis*. Since the act of line-drawing is the process necessary for producing the image of time, it must therefore also be the essential act of all sensible synthesis generally. This fact dictates how we must make intelligible (by exhibiting constructively or “**schematizing**”) Kant’s claim that the unity of the **generic physical object** arises from acts of sensible synthesis determined by **pure concepts** that are originally “functions of unity” in judgment. *If the pure concepts are rules of sensible synthesis, they must also be rules of line-drawing.* That is to say, line-drawing exhibits or schematizes what Kant calls the **synthesis of apprehension**, the primary act of consciously and knowingly bringing the sensible

plurality into certain unified ways-of-difference, whether these are a priori or not. It stands to reason, then, that line-drawing plays a central part in Kant's theory of a priori cognition.

If line-drawing is necessary for schematizing sensible synthesis generally, then it follows that every particular act of synthesis can only be schematized as some *way of inflecting* the act of line-drawing. Different *kinds* of objective unity are produced by employing the act of line-drawing in different *ways*. Now these ways, Kant says, are determined by pure concepts, which are essential "functions of unity" in judgment, and therefore necessary, because judgment is necessary for putting me in epistemic contact with objects of knowledge generally—empirical, logical, or practical. But if functions of unity in judgment are to serve as rules of sensible synthesis, they must be schematized as ways of interpreting the act of line-drawing, i.e., ways by which I exploit line-drawing to consciously emulate the automatic and unconscious acts of sensible synthesis. I must draw a line in different ways in order to bring the pure concepts, as rules of synthesis, into objective reality.

Second thesis: pure concepts must be rules of line-drawing

A priori sensible knowledge is possible for Kant because there are certain a priori features of judgment. These features are non-optional—any possible object of knowledge must be an object cognized through judgment, and so must conform, a priori, to the structure of judgment. When judgment aims at sensibility, its form is articulated "logically" as "**This (S) is P.**" We may call this sensible employment of judgment **ostensive judgment**. My second thesis is that such functions of unity in judgment must act as rules of line-drawing in order to be pure concepts of the generic physical object. In

order to consciously produce instances for the pure concepts, I carry out specially interpreted acts of line-drawing.

Concluding thesis: ostensive judgment is an algebraic relation

From these two theses I will argue for a third: the sensible function of ostensive judgment makes it isomorphic with an algebraic time–state function. The subject-position refers to *substance*, which is time itself as a magnitude. The predicate-position refers to *quality*, which is a continuous magnitude of possible property-instances taken as values on this continuum. *The copula is therefore a relation between two magnitudes, time and state.* The subject- and predicate-positions are thus, in one aspect, identical to algebraic variables. Moreover, they are related so that the subject-position acts as an independent variable whose value determines the value of the predicate-position, which acts as a dependent variable. This determination is itself governed by a *rule*, i.e., it is lawful. Thus, the logical relation of subject and predicate as *condition* and *conditioned* is realized in sensible synthesis (and schematized, as we are doing now) as a lawful sequence of state-values over time. This sequence is lawful because the state-values are *calculated* from their position in time by some discoverable mathematical formula. Thus the natural language grammatical relation “S is P” is rendered in Kant’s system as the archetypal mathematico-physical algebraic relation $t \rightarrow f(t)$.

By Kant’s lights, the grammatical intentions carried out in the act of asserting “This (S) is P” are also physically constructive acts of the imagination. My dissertation will show that these acts of synthesis, when schematized through line-drawing, give rise to the formalism at work in the practice of mathematical physics, which schematizes time and state as lines related by an algebraic formula.

Pure concepts: functions of unity in judgment

The categories are the semantic universals I abstract from my experience of physical objects. They are not, therefore, original presentations, but ones Kant calls “originally acquired.” What are original, rather, are the rules that produce the kinds of unity from which the categories have been abstracted. These are the **pure concepts**—“functions of unity” intended by the effort to assert truth by means of judgment. Specifically, they are *ways of thinking unity* that are *structural* and inherent in the grammatical articulation of the act of judgment. This structure is, generally speaking, “S is P.” I claim that some “presentation,” articulated in the subject-position, “is P.” And by “is P” I mean simply that I know the rule, articulated in the predicate-position, for making instances of a certain *kind*, one of which is identical to the presentation in the subject-position.

“S is P” is the necessary structure of knowledge, and of the knowing consciousness. Thus there is a *structured unity* imposed on the very invocation of the knowing subject in the act of knowing—because knowing depends on the structure “S is P.” The “I think,” which is Kant’s name for the knowing subject *in action*, arises in dependence on the act of assertion, or judgment. The structure of judgment is a priori, and determines what my imagination must do in order to permit knowledge. I know that “S is P” only if I know how to use *P* as a rule for making imaginary instances, and only if one of these is identical to *S*.

We have seen that bringing the “I think” into existence as **apperception**, or the knowing subject, rests on the act of asserting “S is P.” This can be done in reference to two pools of possible contents, or “presentations.” One is the pool of already-abstracted concepts. These are rules of image-making, general kinds that specify a *way of variation*

by which images can differ while still being instances of the same *kind*. The a priori functions of unity in judgment yield the rules of **general logic**.

The other source of reference is the onslaught of immediate presentation produced by the stimulation of my passive power of sensibility. Here, the rules in judgment are ways in which I must imagine the point-moments given in sensation as being interconnected. Specifically, they must be interconnected in just those ways that the components of judgment (subject, predicate, and copula) naturally and automatically intend and pick out. This sensible use of judgment is the topic of Kant's novel **transcendental logic**.

There is a way-of-unity that is the natural (a priori) referent when I assert the subject-position articulated as *this* or *this S*. My assertion of the **subject-position** picks out something about reality, about the passing plurality of point-moments delivered by inner and outer sense. When I intend the referent of the subject-position, I intend to refer to a way-of-unity that I imagine holding between the passing plurality of point-moments given in sensibility—they are linked spatially and temporally. Likewise, the concept in the **predicate-position** picks out another way-of-unity, which is the quality as a dimension of variation, i.e., the general kind of an instance. The predicate-position acts as a rule that forces my imagination to situate an immediate instance within a continuum of differences. This unified continuum is the sensible referent of the predicate-position.

These rules then combine in the copula: the *this* carries a sensibility-spanning unity that is taking as the *substrate* that presents or provides instances of qualities that are rules for producing imaginary instances *of a kind*. When “This (S) is P” is true, I intend that there is a *real connection*, one that holds in reality—and thus independently of my own proclivities for combining concepts in a judgment. What I assert is that a particular “value” of the quality-continuum appears when and where it does necessarily. This is

necessary truth in sensibility. This necessity or lawfulness is conceived by Kant as the determination of the value of the predicate-position (the *property* of the object as a value on a continuum of *quality*) by that of the subject-position (the *substance* of the object which reality-through-time, or time as substrate) in the same way that the dependent (state-value) variable is determined *by calculation* from the independent (time-value) variable in classical mechanics.

In summary: The sensible function of the *predicate-position* is to subsume an empirical content on the continuum of variability which schematizes its subsuming second-order quality; thus the referent of this position is a continuous magnitude, the result of schematizing the apprehension of a **property continuum**, or **quality**, through line-drawing. The sensible function of the *subject-position* is a spatially extended and trans-temporal reality, i.e., a reality that is self-identical through time; thus the referent of the subject-position is also a continuous magnitude, the result of schematizing the apprehension of a real **time continuum**, or **substance**, through line-drawing. The sensible function of the copula is to relate the quality to substance as conditioned to condition.

To cognize the synthetic a priori unity of the general physical object is to think of every point-content in its history as a value within some second-order continuum of *quality* that has been determined, a priori, by its position in the time-continuum of *substance*. Thus the relation of truth in ostensive judgment (the topic of Kant's "transcendental logic") is the determination of one magnitude (the predicate) by another (the subject). And this accords with the logical relation of the subject and predicate in general logic as the relation of *condition* and *conditioned*. This, I will show, is the relation referred to by the semantic category of **causality**. This determination relation in ostensive judgment can be recursive—rules can be rules whose instance are other rules—

and this hierarchical network of general kinds is the web of unity that is the telos of theoretical reason. This also brings Kant's transcendental logic of sensible synthesis, the theory of how the subject, predicate, and copula act as a priori rules inherent to every physical object, into accord with the structures of logical subordination studied of general logic. This relation is not isomorphic—it is limited to the bare structure of judgment as a relation of two unities into one. But this common basis in the S—is—P structure of judgment is enough for Kant to call his exposition of these syntheses of magnitude and their relation through the copula a “**logic**.”

Pure concepts: bases of space, time, and the categories

The pure concepts are functions of unity associated with the components of judgment—subject, predicate, and copula. The components of ostensive judgment are guaranteed to have referents because my acts of synthesis are spontaneous and produce them in imagination. These imaginary referents are then automatically subsumed under the appropriate grammatical positions: *body* and *substance* under the subject-position, *property* and *quality* under the predicate-position, and *causality* through the copula.

Kant's thesis is that the components of judgment subsume the proper aspects of physical unity because these positions also contain the rules that guide my imaginary acts of sensible synthesis. But in order to make sense out of these syntheses, I must be able to carry them out myself—i.e., I must be able to *schematize* them. This is done through an act of line-drawing, performed under a rule for thinking its performance as combining point-moments in certain ways. These ways-of-unity are the necessary features of any physical object. As universal features of all sensible objects, Kant calls them **categories**, following Aristotle. But Kant thinks his list is verifiably complete because it arises from systematic unity, i.e., the structure of judgment.

The functions of unity in judgment are rules that schematize the meaning of the necessary features of all sensible knowledge. These are (1) *space* and *time*, which are pure concepts referring to the dimensions of plurality inherent to the very capacity to sense, called the “formal” intuitions of space and time, and (2) the categories: *magnitude*, *substance*, *property*, *quality*, and *causality*. It is my position that Kant takes line-drawing to be *the* fundamental act of sensible synthesis necessary for cognizing space, time, change, and the unities denoting the generic physical object, whose essential aspects are the Kantian categories listed above.

Space

Line-drawing is necessary for my awareness of **space**. Space is “no cognition at all” prior to line-drawing, and in order to cognize any bit of spatial unity at all, “I must draw it; and hence I must bring about synthetically a determinate combination of the given manifold, so that the unity of this act is at the same time the unity of consciousness (in the concept of a line), and so that an object (a determinate space) is thereby first cognized” [B137–38].

Time

Line-drawing is necessary for my awareness of **time**. “We present time sequence by a line progressing *ad infinitum*, a line in which the manifold constitutes a series of only one dimension. And from the properties of that line we infer all the properties of time, except for the one difference that the parts of the line are simultaneous whereas the parts of time are always sequential” [A33/B49–50]. “The determinations of inner sense must be arranged by us as appearances in time in precisely the same way as the determinations of the outer senses are arranged by us in space” [B156].

Change

Line-drawing is necessary for my awareness of **change**. Not only the time *sequence* but the very meaning of sequentiality (and of passing) is originally constructed through the act of line-drawing:

even time we cannot present except inasmuch as, in *drawing* a straight line (meant to be the externally figurative presentation of time), we attend merely to the act of the manifold's synthesis whereby we successively determine inner sense, and thereby attend to the succession of this determination in inner sense. Indeed, what first produces the concept of succession is motion, taken as act of the subject (rather than as a determination of an object) and consequently as the act whereby we determine *inner sense* according to its form. [B154–55]

We cannot present time “to ourselves except under the image of a line insofar as we draw that line; without exhibiting time in this way, we could not cognize the singleness of its dimension” [B156].

There must be a purely mathematical theory of the categories

The subject- and predicate-positions in “This (S) is P” not only subsume certain non-empirical ways-of-unity, but also construct them. Kant says that the **subject-position** (*this S*) contains two functions of unity, which are the rules of **magnitude** and **substance**, and so a priori subsumes these products of automatic synthesis. To schematize these—to make them consciously—I draw a line under a certain interpretation. I schematize magnitude or *body* by moving a point, posited in imitation of reality, while attending to its identity across space, which yields an imaginary cognition of reality as being extended across space. I schematize *substance* by moving a point while attending to its identity across time, which yields an imaginary cognition of reality as being extended through time. The **predicate-position** (*P*) contains two functions of unity, which are the rules of **property** and **quality**. To schematize *quality*, I draw a line intended as a continuum of variations that all fall under the same concept. For example, I schematize *red* as a

spectrum of incrementally varying hues, bounded by orange-red and violet-red. A property is a momentary instantiation of one of these hues, which is a value on a continuum, constructed through line-drawing, which Kant calls “intensive magnitude.”

The subject- and predicate-positions in ostensive judgment both refer to what is, structurally, the act of presenting an imaginary line. But this is also the construction of a *magnitude*. The subject-position is schematized as reality in spatial and temporal extension. Reality that is extended as spatial magnitude is a *body*, and reality that is extended in temporal magnitude is a *substance*. These are the aspects of synthesis referred to by the subject-position, and they are schematized by drawing a line, taken as the positing of reality as a moving point, across space and time, respectively. The predicate-position is schematized as a continuum of variation wherein different instances all fall under the same *general kind*. Thus Kant makes not only *substance* but also quality into a magnitude. A *property* is a momentary instance in this continuum, and thus also a particular value in it.

The interesting result, undetected in previous Kant scholarship, is that the relation of subject and predicate in ostensive judgment is schematized as a relation between two imaginary *magnitudes*. This relation is the sensible function of unity in the **copula**. This relation subsumes (and constructs) objective truth—a property is what it is at time *t* by necessity, i.e., independently of psychological association or concept analysis. This is how Kant schematizes synthetic a priori truth, which is the official task of his positive program. The necessary but non-analytic relation of objective truth is actually the lawful determination of the sequence of properties, belonging under the same second-order quality, as numeric values that are determined by their position in time (another numeric value). That is, the relation of physical or “synthetic a priori” truth is a relation of *mathematical determination*, i.e., one where the value of one variable (the present

position of a point-moment in the substrate of time, which is how Kant defines *substance*) determines another (the empirical content filling that point-moment as a value in the continuum of variability within the same *kind*, which is how Kant defines how *property* is unified under a *quality*). The copula is thus the function of unity determining the sensible synthesis of **causality**. This determination is lawful. A content is real simply by being empirically given, but a *relation* is real when it is determined by an unwavering law. Kant thus conceives of the functions of unity in ostensive judgment as two magnitudes related so that the value of one determines the value of the other according to some intelligible and mathematical law. But this is exactly what is thought in the relation between independent and dependent variables in an **algebraic function** of the form $t \rightarrow f(t)$.

In my dissertation I will show that, because line-drawing is the necessary vehicle of all pure apprehension, the functions of **intellectual synthesis** at work in “This (S) is P” are schematized as rules of **sensible synthesis** that are structurally identical with the determination of the value of one variable (subsuming state) by another (subsuming time) by an algebraic function of the form $t \rightarrow f(t)$.

KANT’S LINE-DRAWING PASSAGES

Transcendental Aesthetic

As early as the **Transcendental Aesthetic**, at the very opening of the body of the First *Critique*, Kant says that self-intuition alone (i.e., without understanding and synthesis) only presents a flow of inner states. This pure flow is something that I undergo passively but without a continuing sense of “I.” The continuing unitary “I” that transcends the stream of constantly novel sensations, Kant says, is not a particular but a

thought, that is, an act of combination. Taken in isolation from understanding, whose job is combination or synthesis, the flow of sensibility is unintelligible.

It is interesting how Kant characterizes this unintelligibility of unitary time (the objective correlate of the lack of apperception, or consciousness of consciousness as the subject of knowledge): he says the unconnected flow of sensible states “gives us no shape”; that is to say, it lacks *unity in its presentation as an intuition*. Unity in intuition can only be spatial for Kant; he identifies *sensible* unity with *spatial* unity. Time may be the fundamental form of the connected (and thereby transcended) flow of sensible unity, but it is not intelligible until it has itself been *spatialized*:

And precisely because this inner intuition gives us no shape, do we try to make up for this deficiency by means of analogies. We present time sequence by a line progressing *ad infinitum*, a line in which the manifold constitutes a series of only one dimension. And from the properties of that line we infer all the properties of time, except for the one difference that the parts of the line are simultaneous whereas the parts of time are always sequential. This fact, moreover, that all relations of time can be expressed by means of outer intuition, shows that the presentation of time is itself intuition. [A33/B49–50]

From the properties of space (that is, of the drawn line) we infer “all the properties of time.” We see how important this really is when Kant brings it up repeatedly throughout the remainder of the First *Critique*. Line-drawing is the archetypal act of the threefold synthesis that is the central doctrine of the Transcendental Deduction. He makes four references to it in the A edition (1781) and seven in the B edition (1787) of the *Critique*. In the Schematism, the production of time is identified with the successive act of apprehending any spatial and temporal manifold. In the Systematic Presentation, the principles of magnitude, quality, substance, and causality are each presented as ways of line-drawing.

A Deduction

In the **A Deduction**, line-drawing is how Kant explains the threefold synthesis of apprehension, reproduction, and recognition. Producing a unified plurality that is spatial, temporal, or finally numerical requires three distinguishable steps, requiring minimally that I “apprehend in thought one of these manifold presentations after the other” [A102]. Through Kant’s examples, it is clear that the feared loss would be that of “past” points posited through line-drawing. This would be a failure of **apprehension** (the collecting of a plurality into a known unity) and also a failure to “bring” the past into the present *as presentation* that is both intuitive (i.e., as a line) and conceptual (i.e., as a thought-unity that subsumes the line from the position of consciousness). Past moments must be **reproduced** into unity with present moments by presenting them *together* with the present moment *as points in space*. The past is retained through an act of line-drawing interpreted as an act that brings past moments into compresence with the present moment as many points together in one space. The past is thereby continually “reproduced,” and this cumulative reproduction is **recognized** or “thought” as a unity, a unitary *this*—even though its referent is a continually extending apprehension of plurality.

The thought-unity of recognition, Kant says, is actually the unity of a rule that I use to emulate this threefold synthesis myself, consciously. I will show that Kant believes that this emulation of sensible synthesis under a pure concept has the act of line-drawing as its universal or generic form, and that the pure concept serves as a rule of sensible synthesis by specifying how the act of line-drawing should be interpreted.

The pure concept is originally a rule of “intellectual” synthesis—i.e., a rule of term-combination in judgment. It rests in the understanding as an innate structural entity that Kant calls a “function of unity” in judgment. The structure of judgment is “S is P.” When the terms in judgment (i.e., the subject- and predicate-positions) are filled with

already-abstracted concepts, such as universals and names, these functions behave like the logical operators of general logic.

Kant's novel thesis is that the functions of unity underlying the logical forms of concept-relation also serve as the functions that combine sensations into the unity of the physical world. Because these functions are necessary for judgment, they are also necessary for cognition, or sensible "apperception." The subject of sensible knowledge thus has the structure of judgment as its unity. We may call judgment that aims towards sensibility (instead of towards the store of already-abstracted concepts) *ostensive judgment*, and its structure, accordingly, is "This (S) is P."

In ostensive judgment, the pure concepts are functions of unity that act as rules of "sensible" synthesis. I will argue that sensible synthesis can only be carried out consciously, or schematized, as an act of line-drawing. This is because time is originally generated through line-drawing. But this means that the pure concepts, which function sensibly as rules of "transcendental time determination," can only be schematized, and thereby rendered intelligible, as rules of line-drawing. A rule in this case is a particular way of specifying how the act of drawing is to be "thought," i.e., the way in which the connection (reproduction) of its constituent point-moments is to be understood. The pure concept thus tells me how to use line-drawing to construct a certain kind of semantic content. This is the significance that is "thought" in the semantic version of the pure concept, or **category**.

The rule pervades the line and unifies it over and above the unity provided in intuition, and so it specifies the *kind* of unity I should invent as I draw the line. This is the unity that links the point-moments into what Kant calls *recognition in the concept*. This recognition is the awareness of the rule I am using to produce the kind of point-moment connection at issue. For example, when I draw a line by moving a point that I imagine to

be numerically identical despite the passing of time, I invent the sense of *substance*. A pure concept is a way for “thinking” the unity line-drawing by intending a certain kind of link between the elements of my activity, which is one of “producing” and “adding” a series of point-moments “little by little” [A103].

Thus there are two components of the threefold synthesis. The unity of the act is *presented in intuition* as the unity of the line: the original sensible plurality-in-unity. But this unity is “thought” at an even higher level—the unity of the rule (recognition) I use to produce a certain kind of connection between point-moments (reproduction). This is how the unity of the act is *presented in understanding*.

I will show that this unity, in part, must be characterized as an algebraic variable, which ranges over the unity of the line. This is how pure sensible synthesis is “thought” under a term—a subject- or predicate-position in ostensive judgment. A line is *always* a magnitude, no matter what particular grammatical sense it is given by the rule at work. Thus the subject- and predicate-positions are, in part, algebraic variables, the only difference being that what they refer to is not merely a line, but a line interpreted in a certain way—a line combined under a certain grammatical interpretation that produces one of the essential sensible syntheses imagined to be in the generic physical object, and which is then abstracted as a category.

B Deduction

In the **B Deduction**, Kant says explicitly that time can only be presented by acting out by mean of apprehension, which is temporal in its gathering but also unifying. This act, he says, is that of “motion taken as the *describing* of a space” [B155 n. 283]. But moving a point through space in order to “describe” it is nothing other than line-drawing. We cannot “present” time to ourselves, Kant says, “except under the image of a line

insofar as we draw that line; without exhibiting time in this way, we could not cognize the singleness of its dimension” [B156]. Consequently, “the determinations of inner sense must be arranged by us as appearances in time in precisely the same way as the determinations of the outer senses are arranged by us in space” [B156]. The unity of time is isomorphic with the unity of space—of a line. Thus all pure concepts, all rules of intellectual synthesis, which Kant identifies as rules of the subject- and predicate-positions, and rules for their subsequent combination in the copula, are rules operating on the act of line-drawing.

The domain being synthesized spontaneously is that of inner sense—of time. Time contains space (or: the change of passing-away pervades space), and it also contains my own unity as a knower that contains an empirical content. How is time synthesized consciously, i.e., schematized? Kant tells us:

And even time we cannot present except inasmuch as, in *drawing* a straight line (meant to be the externally figurative presentation of time), we attend merely to the act of the manifold’s synthesis whereby we successively determine inner sense, and thereby attend to the succession of this determination in inner sense. [B154]

The rule that “thinks” or “recognizes” the line as *time* is the command: “Attend to passing while you move the point, in emulation of it. This motion of a *unitary* point transcends passing. This line now presents identity-through-passing, which is time.”

There is no time *as object* except after we make it. An object that contains (spatiotemporal) combination in its essence, as sensible objects do, can only be recognized under a rule of emulation if this combination is also emulated. This is the meaning of sensible unity: combination. For a combination to have meaning, i.e., in order for me to know what such combination is, I have to carry it out myself. This is Kant’s principle of *meaning by way of schematism*. This combination is internal to the presented

object; it can be internal to my understanding only if I can make it. Making a plurality into a unity *in sensibility* can only be presented spatially. Thus, Kant says, “as regards time, which after all is not an object of outer intuition at all, we cannot present it to ourselves except under the image of a line insofar as we draw that line; without exhibiting time in this way, we could not cognize the singleness of its dimension” [B156].

Schematism

In the Schematism, Kant says that the schema of magnitude “contains and is responsible for the presentation of ... the production (synthesis) of time itself in the successive apprehension of an object” [A145/B184]. Time, he goes on to say, is the very “image” of magnitude. Time itself, that is, as object of apperceptive consciousness, whose unity is that of a knower that knows through “S is P,” is a line of magnitude. We learn that the “schema of substance is permanence of the real in time,” that this real is nothing but a permanent framework or substrate that holds time-positions in place, a priori of their being filled with this or that state-value, and that the real point of this construction is the permanence of time itself: “Time is not in transition; rather, the existence of what is mutable is in transition in time” [A144/B183]. Substance is time, time is a line, and this is the schema of the grammatical subject. The subject-position refers to a line.

Systematic Presentation

Finally, in the Systematic Presentation, where Kant presents his principles of physics, line-drawing is the basis of every principle of physical reality.

In the Axioms of Intuition, we are told that producing the objective space of Euclidean geometry and of the objective time of physics (and clocks) is the result of line-drawing:

I can present no line, no matter how small, without drawing it in thought, i.e., without producing from one point onward all the parts little by little and thereby tracing this intuition in the first place. And the situation is the same with every time, even the smallest. In any such time I think only the successive progression from one instant to the next, where through all the parts of time and their addition a determinate time magnitude is finally produced. [A162–3/B203]

In the Anticipations of Perception, we are told that “any reality contained in appearance has intensive magnitude, i.e., a degree” [A168/B210]. The *sense* of this degree is schematized by drawing a line from the “intensity” of the second-order quality “down” to its utter negation. “In other words, the real contained in appearance has always a magnitude” [A168/B210]. This distance is constructed, again, through line-drawing: “every reality has its degree, which can decrease to nothing (i.e., emptiness) by infinitely many steps, with the extensive magnitude of the appearance being unchanged” [A172/B214].

In the Analogies of Experience, we are told that permanence, the referent of the subject-position, “expresses time as such,” which means that the subject-position refers to magnitude in the form of a line. State is also a magnitude, as we have mentioned. The relation of subject and predicate is that between two magnitudes, which Kant in the Second Analogy calls *time* and *change*:

Now every change has a cause that manifests its causality in the entire time wherein the change takes place. Hence this cause produces its change not suddenly (i.e., all at once, or in one instant), but in a time; so that, as the time increases from its initial instant (*a*) up to its completion (*b*), the reality’s magnitude ($b - a$) is also produced through all the smaller degrees contained between the first degree and the last. [A208/B253–54]

From these passages we can see that, far from being a mere metaphor or illustration of the act of synthesis, Kant takes line-drawing to be the essential universal act of synthesis. Specifically, I will argue that line-drawing is the generic activity that each of the categories depends on for its sense. Line-drawing is the act that schematizes each of the forms of judgment to give it its sensible (imaginary) significance.

While some commentators recognize the importance of line-drawing to the construction of space, and a few take Kant at his word regarding its importance to the very production of time, none have seen it as the essential act behind the schematism and production of the Kantian categories. But this is exactly what it is. When we realize this, however, some important long-standing problems are elegantly solved:

- Q: How is it that combination of presentations (terms) in judgment can determine the combination of presentations (point-moments) in intuition?
A: The grammatical (subject and predicate) positions in judgment function precisely as algebraic variables ranging over continuous magnitudes, lines, or real number continuums. The unity of each term is the unity of a line, and their combination is the unity of an algebraic function.
- Q: What is the point and role of the Schematism chapter?
A: To show how line-drawing provides the interface between pure rule and sensible synthesis.
- Q: What is the problem that vexed Kant and propelled him to rewrite the First *Critique*?
A: Line-drawing is prior to both (formal) space and time. Each is dependent on the other. Time is the more inclusive container, but space is the only way to make plurality-in-unity, and thus synthesis, intelligible.

THE PRIMARY PUZZLE OF THE TRANSCENDENTAL ANALYTIC: HOW CAN TERM COMBINATION CONTROL PHYSICAL OBJECT CREATION?

We have seen that the unconscious acts of spontaneous synthesis, in order to be “recognized” in a concept, must be consciously carried out, since each categorical meaning is a way of grammatical combination, and meaning is knowing the rule for producing an imaginary instance oneself. This conscious act, Kant has said, is carried out by drawing a line while attending to the kind of combination demanded by a grammatical position, and then these combinations are combined yet again through the copula. The terms of judgment are the functions of unity “thought” in the subject- and predicate-positions, and these range over continuous magnitudes, or lines, in the same way as algebraic variables. This is how a term of judgment acts as a unitary “rule” over a sensible plurality: it groups it together into a line, and this line is “thought under” the variable. By moving a point, and producing a line, I produce the sense of the variable—it is always some way of linking the passing of inner sense, now reproduced as a spatial line of magnitude.

But Kant does not tell us this when he opens the Transcendental Logic, which treats the function of intellectual rules when applied to sensibility. Instead, he presents a Table of Judgments, which contains forms similar to those of general logic, which deals only with the combination and analysis of *universals*.

Kant’s central thesis is that the functions of unity at work in *judgment* are also functions of unity at work in *intuition*, when intuition is cognized through ostensive judgment. This is how he implements his Copernican hypothesis that ontology conforms to logic. But how can the unity of terms in judgment have anything to do with the unity of point-moments imagined to be in a physical object? Kant’s initial answer is simply that the rules of discursive synthesis, which yield effects that are studied in general logic, also

serve as rules of sensible synthesis, which yield the unity of the generic physical object, whose corresponding aspects are abstracted as the *categories*. In order for Kant's theory to work, the functions of unity at work in judgment *must* also serve as the functions of unity at work in intuition. This is how Kant introduces his revolutionary idea in the Metaphysical Deduction:

Bringing various presentations *under* a concept (a task dealt with by general logic) is done analytically. But bringing, not presentations but the pure synthesis of presentations, *to* concepts is what transcendental logic teaches. The first [thing] that we must be given a priori in order to cognize any object is the *manifold* of pure intuition. The second [thing] is the *synthesis* of this manifold by the imagination. But this synthesis does not yet yield cognition. The third [thing we need] in order to cognize an object that we encounter is the concepts which give *unity* to this pure synthesis and which consist solely in the presentation of this necessary synthetic unity. And these concepts rest on the understanding.

The same function that gives unity to the various presentations *in a judgment* also gives unity to the mere synthesis of various presentations *in an intuition*. This unity—speaking generally—is called pure concept of understanding. Hence the same understanding—and indeed through the same acts whereby it brought about, in concepts, the logical form of a judgment by means of analytic unity—also brings into its presentations a transcendental content, by means of the synthetic unity of the manifold in intuition as such; and because of this, these presentations are called pure concepts of understanding applying a priori to objects. Bringing such a transcendental content into these presentations is something that general logic cannot accomplish.

Thus there arise precisely as many pure concepts of understanding applying a priori to objects of intuition as such, as in the preceding table there were logical functions involved in all possible judgments. [A79/B104–5]

We can call this the **double function** passage: the unity of terms and their combination in judgment “also gives unity to the mere synthesis of various presentations *in an intuition*.” Kant is saying that the intellectual (term-combining) and sensible (point-moment combining) functions of unity are the same. For Kant, the grammatical positions refer to ways of combination. The function of unity that I think in the subject-position is

the same as the function of unity used to construct its referent in the imagination. The same holds for the predicate-position. And the same holds for the copula. This means, minimally, that there must be a correspondence between the components of judgment (subject, predicate, and copula) and the necessary features of the generic physical object.

All commentators recognize the importance of the double function passage, but none have been able to take it at face value and show how the double function thesis is possible. How can function of term-combination be isomorphic with a function of sensible synthesis?

Longuenesse's interpretation of the double function passage

Very few commentators have been able to make sense of anything like a literal interpretation of Kant's double function theory of the pure concepts. The closest thing to a literal interpretation of Kant's claim of a direct determination of sensible synthesis by the *relation of terms* in judgment comes from Beatrice Longuenesse.¹ Instead of showing a direct relation between grammatical position and imaginary feature, however, she only shows that the acts of synthesis must finally produce things (individual physical objects) that can be compared, reflected upon, and abstracted from, in order to permit the generation of universals, which is accomplished by making analytic judgments.

Longuenesse agrees that Kant's notion of logical form is not that of the modern truth-functional operator, but something more basic, i.e., forms of those mental activities that are "necessary for any representation of an object" (Longuenesse, *Capacity* 5). And she rightly argues that the anti-psychologistic readings of Cohen, Heidegger, and Strawson must fail, because Kant's theory cannot be even *understood* without recourse to mental activity as an explanatory ground:

¹ Béatrice Longuenesse, *Kant and the Capacity to Judge: Sensibility and Discursivity in the Transcendental Analytic of the Critique of Pure Reason* (Princeton: Princeton University Press, 1998).

both in the Transcendental Deduction of the Categories and in the Analytic of Principles, which the Deduction is meant to ground, Kant's argument for the applicability of categories to objects rests on the relation he tries to establish between discursive syntheses or combinations (combinations of concepts in judgments) on the one hand, and syntheses or combinations of our sensible perceptions on the other. Such an argument is undeniably "mentalist" or "psychological." (Longuenesse, *Capacity* 6)

What is lacking in Kant scholarship, she says, is a "systematic investigation of the relation between logical functions of judgment and categories, and of the import of this correlation for Kant's principles of pure understanding. Such an investigation is what I am presenting in this book." (Longuenesse, *Capacity* 6–7)

Longuenesse's thesis (and the title of her book) comes from Kant's declaration: "Now since all acts of the understanding can be reduced to judgments, the understanding as such can be presented as a *power of judgment* [*Vermögen zu urteilen*]" [A69/B94]. And again: "This division of the categories has been generated systematically from a common principle, viz., our ability to judge (which is equivalent to our ability to think [*Vermögen zu denken*])" [A81/B106].

In the *Metaphysical Foundations of Natural Science*, Kant promises to reformulate the Transcendental Deduction of the categories by deducing it, in the B edition of the *Critique*, from the "precisely determined definition of a judgment in general." This is the point of § 19 in the re-written Deduction, entitled "The logical form of all judgments consists in the objective unity of apperception of the concepts contained in them," which states that "a judgment is nothing but a way of bringing given cognitions to the objective unity of apperception" [B141].

There are two ways in which discursive thought relates to what is given in sensibility: (1) how we form universals from sensible objects (i.e., how we "reflect" them under universals), and (2) how we first generate sensible objects that *can* be reflected under universals. Only (2) is revolutionary. (1) is not developed in the First *Critique*

because Kant saw it as obvious: reflection requires the “logical” functions of unity in judgment. The process of reflection is performed “*with a view to forming judgments.*” (Longuenesse, *Capacity* 11)

Longuenesse says we cannot understand the role of the forms of judgment as a “guide” for the table of categories unless we understand their role in the process of reflection. The forms of judgment are *nothing but* the forms of “comparison, reflection, and abstraction.” If we consider the function of these forms as forms of reflection (analysis), this will tell us the kinds of synthesis that are required in order for this to occur:

consider the forms of the analysis of what is given in sensibility (the forms of ‘comparison, abstraction, reflection’ —the logical forms of judgment) and you will have the key to the forms of the *synthesis that must occur prior to analysis*, namely the synthesis required for the sensible representation of the *x*’s that can be reflected under concepts according to the logical forms of our judgments. (Longuenesse, *Capacity* 11)

The forms of judgment are guides in the sense of being the final goals of synthesis, not in the sense of being rules of the actual procedure of synthesis. For her, the Table of Judgments is a guide only because the sense-world finally must be digestible according to the truth-functional rules of logic. This is how she understands Kant’s thesis that the sense-world conforms to the structure of judgment. She does not understand it as an informing structure in its own right.

According to Longuenesse, Kant’s key idea is that the use of concepts in judgments, which is studied by general logic, depends on a prior sensible synthesis; otherwise, the sensible manifold could not be thought under (ordinary empirical) concepts. It is only in this weak sense the sensible synthesis must be “guided by” the forms of judgment, and this is her interpretation of Kant’s claim that the same function

that gives unity to presentations in a judgment also gives unity to their synthesis in an intuition.

For Longuenesse, the sensible manifold must be *prepared* for the application of the structure of judgment. The manifold must ultimately be *amenable* to digestion by the subject-position, the predicate-position, and the copula. But she does not go further and show how these same elements themselves, as functions of term-thinking and term-combination, directly relate to these sensible syntheses. The combining that goes on in the empty placeholders of judgment as I exert my effort to speak truth about sensation is something that, for Longuenesse, cannot literally be mapped onto the structure of judgment, “This (S) is P.” Sensible synthesis is not directly determined by the structure of judgment, but is rather the result of other and independent acts of sensible synthesis that are “guided” by this structure as a *telos*—syntheses that must occur in order for ostensive judgment to have sensible reference. The sense-world yields physical objects that are isomorphic with the structure of judgment (and thereby general logic), but not by direct application of this structure. Longuenesse does not show, or try to show, that the sensible syntheses constructing the generic physical object reflect the very rules contained in, and thought through, the very structure of judgment and its forms.

So although she does not interpret Kant’s “logical” functions of unity as forms of general logic, Longuenesse still does not see them as constitutive *of* synthesis, only as end-points *to which* synthesis must conform. The key difference is in notion of *governance*:

our motto should be: use the forms of analysis (the logical functions of judgment) as your guiding thread to the “universal representations of synthesis” (the categories). This is because synthesis of what is given in sensibility is achieved in order to make analysis possible. Categories before synthesis are nothing but mere forms of analysis, logical functions of judgment. But these “mere forms of

analysis” govern the synthesis of what they are to analyze. (Longuenesse, *Capacity* 12)

Like Longuenesse, I agree that the categories can only be adequately understood at their root—i.e., in the functions of unity in judgment. But she separates these forms from the acts of synthesis. That is, synthesis is “guided” by these forms only insofar as the results of synthesis must be capable of analysis by these forms. I, however, argue that the forms of judgment are guides in the direct sense.

My interpretation of the double function

I will defend a more literal interpretation: that there is a direct correspondence between the sensible syntheses in imagination and the terms of judgment, and that the copula combines these syntheses into the higher “objective” unity of the generic physical object. Unlike Longuenesse, I will not argue that the structure of judgment merely serves as a “guide” by providing the final unity to which various occult and unintelligible acts of sensible synthesis must ultimately conform in order to make judgment and general logic possible. Rather, I will argue that structure of judgment is at work directly, providing rules for sensible synthesis that *correspond to* that structure. When I intend the subject-position, predicate-position, and copula, there are combinations matching their intentionality.

I will show that Kant’s own explanations clearly indicate that the grammatical positions of judgment function exactly as algebraic variables that range over real number continuums, or *lines*. When I “intend” an algebraic variable x , I intend, in a single “thought,” and in the guise of a “term,” a unity that ranges over a plurality—i.e., to the unity that I create through line-drawing.

Dismissive interpretations of the double function

Most interpreters simply dismiss Kant's central thesis. As Longuenesse mentions, Cohen, Heidegger, and Strawson see any reliance on mental acts as already a failure in method, since mental activity is separable from epistemic warrant, which is what Kant is really after. But there is another common reason for rejecting Kant's thesis. This is the fact that the forms of judgment appear to be identical to the merely truth-functional operators of contemporary formal logic.

Herman Cohen saw the very idea of a metaphysical deduction as irrelevant to the discovery of the categories, the true source of which was Newtonian science. He therefore proposed reading the Transcendental Analytic backwards, from the Systematic Presentation, which presents principles of physics.⁸

Strawson notes that only primitive logical forms could plausibly be called necessary. For example, it includes both hypothetical and disjunctive forms, when these are really interdefinable with the help of negation: "It is not enough that these are forms which the logician can frame, or even forms which we in fact use. For if the form is derivative, then any pure concept the use of which is involved in the use of the form is derivative also and hence not a category."⁹

Perhaps the strongest condemnation of Kant's double function theory comes from T. K. Seung, in his "Kant's Conception of the Categories."¹⁰ The decisive feature of Seung's interpretation of A79/B104–5 is his characterization of the logical/real distinction as one that is drawn *within* the discursive domain of concepts. To be sure, Seung recognizes that the real categories depend on the intuitional domain for their

⁸ Herman Cohen, *Kants Theorie Der Erfahrung* (Berlin: B. Cassirer, 1918) 345–46.

⁹ Peter F. Strawson, *The Bounds of Sense: An Essay on Kant's Critique of Pure Reason* (New York: Routledge, 1966) 80.

¹⁰ T. K. Seung, "Kant's Conception of the Categories." *The Review of Metaphysics* 43.1 (1989) 107–132.

possibility: “Logical categories develop into real categories in the domain of intuitions.” (Seung, “Kant’s Conception” 111) But the importance of this domain-dependence is immediately dropped, and Seung treats the logical/real distinction as grammatical for the remainder of his analysis. Seung reads the distinction not as arising from the difference in domain-application of a single function, but as an *inherent* difference between two types of discursive entity. Two paragraphs later, Seung says that the demarcation between discursive and intuitive expressions “can be captured by the modern logician’s distinction between logical and descriptive terms,” and concludes in the following paragraph that the relation between Kant’s two functions “is roughly the same as that of syntactic and semantic terms in contemporary linguistic terminology.” (Seung, “Kant’s Conception” 111–12) All of this assumes that the forms of transcendental logic are functions for combining already-abstracted universals, and this assumption (as we will see) is wrong.

It is quite correct that the operators of general logic cannot serve as functions of unity for combining the plurality of passing point-data into physical objects. Because of this, most commentators abandon any hope of making good on Kant’s claim that functions of term-combination in judgment also serve as functions combining point-data into physical objects.

I say that the “function that gives unity to the various presentations *in a judgment*” that Kant refers to is the combination through the structure of judgment, “S is P,” and that this structured relation expresses the unity of “the mere synthesis of various presentations *in an intuition*.” However, while term-combination and mere synthesis share one and the same structuring relation, this relation is *not* that of the logical subordination of one universal under another. Rather, the relation is one between magnitudes: the continuum of *time* is thought under the subject-position, the continuum of quantified *quality* is

thought under the predicate-position, and therefore their relation through the copula is a relation between two magnitudes.

Seung and Strawson

To be sure, there is some analogy between the forms of transcendental logic, which are forms for combining point-moments, and the forms of general logic, which are forms for analyzing physical objects into universals and then recombining them into syllogism-ready non-ostensive judgments. Commentators such as Seung stress the analogy over the distinction and hold that Kant actually does identify the pure concepts as forms of general logic. But since forms of general logic cannot determine the unity of appearances into cognition of physical objects, this being the primary function of the pure concepts, interpreting the pure concepts as forms of general logic has the effect of undermining the entire edifice of Kantian metaphysics at its very foundation. If Kant's object-constructing forms are taken as truth-functional operators, then the entire positive program of the First *Critique*—the program of a priori objective knowledge which he says conforms to and derives from a priori forms of unity in judgment—loses all validity.

One problem with this interpretation is that there is plenty of textual evidence against it. Although Kant surely strives to align his transcendental logic with general logic at the beginning of the *Analytic*, later on he develops new models of the pure concepts that have no apparent relation to the judgment-forms of the *Metaphysical Deduction*. These Kant conspicuously discards, along with what used to be their corresponding categories. It is only later in the *Analytic* that Kant develops the actually functioning categories that he describes in the *Systematic Presentation* through his principles of physics, so his later use of the term “logical” must refer to a different (although largely unarticulated) notion of “unity in judgments.” It is my purpose to

articulate this actually functioning unity in detail precisely in its relation to the structure of judgment.

Another problem with interpreting the pure concepts as forms of general logic is that it discourages investigation of these later arguments. These later arguments allegedly describe the pure concepts. But if the pure concepts have been interpreted as forms of general logic, these later sections should *ipso facto* be discarded, no matter how promising they might seem apart from their link to the forms presented in the Metaphysical Deduction.

Other commentators, such as P. F. Strawson, stress the distinction over the analogy and hold that the pure concepts can and should be entirely detached from their previously announced relation to judgment-forms. Strawson claims that there *are* pure concepts—that is, he claims that there are certain ways experience must be conceived by us in order for experience to be *experience*, and these ways are referred to by the concepts of *space*, *substance*, and *causality*. But these concepts have nothing to do with judgment, except that they involve the a priori distinction between subject and predicate:

The excursion through the forms of logic has not advanced us a single step. We are left merely with the notion of unschematized categories, if any, corresponding to the logical distinction of individual ‘name’ (definite referring expression) and predicate-expression. Referring this logical distinction to the conditions of making objective judgments of experience seems to give us at most the notions of particular object and universal kind or character as ‘categories’ which must have application in a world in which such judgments can be made. But this meager result we might have attained directly from the original distinction between intuitions are concept, sensibility and understanding. (Strawson 82)

Transcendental logic is not a logic of universals

The dismissive interpreter makes the mistake of interpreting the “functions of unity in judgment” that are supposed to also be functions of sensible synthesis with the familiar truth functions of general logic. But Kant himself says that the two logics are

distinct. General logic studies how the forms of judgment and inference operate on concepts *that have already been abstracted*. General logic, Kant says, “expects presentations to be given to it from somewhere else—no matter where—in order then to transform these presentations into concepts in the first place” [A76/B102]. To understand the difference between the two logics, we must distinguish the *two domains* to which the subject- and predicate-positions can refer: general logic studies judgment whose grammatical positions refer to *already-abstracted concepts*, while transcendental logic studies ostensive judgment, whose grammatical positions refer to the plurality of passing point-moments given through outer sense.

In general logic, the presentation that falls under the subject- or predicate-position must be an *already-abstracted concept*, what Kant calls a “universal (*repraesentatio per notas communes*) or reflected representation (*repraesentatio discursiva*)” [JL § 1]. We can call these **reflected concepts**. A reflected concept is a rule for image making that has been learned from experience, through the process of reflection.

The *pure* concepts studied by transcendental logic, on the other hand, are not originally reflected concepts and thus not even predicates or indeed anything appearing as a semantic content filling the subject- or predicate-position. Rather, Kant says, they are rules of combination—functions of unity that combine point-moments into the unity of the generic physical object. While reflected concepts are rules that I abstract from experience and then use to make *possible* images (i.e., ones whose instances are spatially extended and can be presented in a moment), the pure concepts are not rules for making images, but for guiding my unconscious acts of point-moment apprehension. Ordinary concepts are rules for making images, but pure concepts are rules for *enacting* kinds of pure synthesis—which I must schematize through acts of line-drawing.

Despite Kant calling them both *logics*, general logic and transcendental logic are distinct. General logic deals with universals and their relation in truth. Specifically, as outlined in the *Jäsche Logic*, general logic deals with (1) the generation of reflected concepts, (2) the truth-functional operations on judgments that relate reflected concepts, and (3) rules of inference between such judgments. As a result, general logic treats the unity of the subject and predicate-positions as the unity of reflected concepts, and their subsequent unity through the copula as a containment relation due to shared semantic contents. In “All red triangles are red,” I know the rules for schematizing the subject-universals, *triangle* and *red*, and so construct a red triangle in imagination. I know the rule for the predicate, which is also *red*, and realize that by this rule I can schematize a red instance that is homogeneous with the red of the triangle. I recognize that the predicate universal subsumes the subject universal, that the class of all red things contains all red triangles, and in this I recognize truth. In “Some cats are black,” I schematize the meaning of some by constructing a Venn diagram showing that the subject and predicate classes overlap. This is the meaning of *some*, and again I recognize truth.

Transcendental logic is completely different, except that it treats the same basic structure of judgment as general logic, i.e., “S is P.” Unlike general logic, transcendental logic has nothing to do with the generation or relation of universals. Rather, it deals with the generation of meaning (and objective unity) through grammatically-guided acts of “transcendental synthesis.” By transcendental synthesis Kant means acts of combining pure point-moments—empty positions in space and time that can hold point-events. Pure synthesis, I will show, is fundamentally an act of line-drawing. To do this, I will extend the explicit analysis that Kant uses in his explanation of time.

General logic and transcendental logic are distinct

General logic and transcendental logic are distinct. General logic points to the realm of universals and produces logical subordination. Transcendental logic points directly to the passing plurality of outer sense and produces what he calls the “transcendental content” of the physical object—the content of the synthesis of point-moments. One has the meaning of a rule as its content; the other, the activity of pure synthesis.

By contrast with general logic, which treats subjects and predicates only as universals, transcendental logic, on the other hand, deals with the generation of ways of pure synthesis. Concepts can refer not only to contents, but also to ways of combining sense contents according to their a priori spatial and temporal separation. Sensibility delivers a plurality of sense-consciousnesses, or point-moments. These are then actively imagined as being connected with one another *across* space and time. Data are originally given in separation, but are then imagined as being connected despite this. These are imaginary connections. Finally, the way-of-connection can be “thought” as a rule—i.e., the rule for carrying that *type* of connection. This type is the referent of a universal, which can be generated through the ordinary process of reflection.

Sometimes we imagine connections according to some arbitrary and conscious plan. But some connections are necessary and unconscious—that is, they are carried out *spontaneously*. There are thus necessary objective connections. These are the connections that unite the passing plurality of point-data, which we can imagine as momentary “pixels” (on analogy with the contents of a computer animation), into what Kant calls the *transcendental object* and which we understand to be the *generic physical object*. There is a kind of unity that all physical objects share, and this unity is necessary and essential to the physical object. We cognize the physical object as a kind of lawfulness. These laws

are our own—they are the way we must imagine passing point-data in order to *know* or *experience* a sense-world. To know or experience has an essential form: I am only conscious of objects *about which I can claim consciousness*. To be conscious is to be able to emulate: to produce an imaginary instance of a given instance *as an instance of a kind*. I know by asserting the kind of an instance. Asserting is constructive: I make the object to verify that I know the rule, which I name, thereby creating a *universal*. I think: “This is *P*.” This is what I do when I am conscious of being conscious—I *intentionally assert* something, and this verifies my existence of an “I” that thinks (and knows) the *this* as a *P*. Do I really “know” that *this* is a *P*? To check, I follow a procedure, or rule, that limits my imaginary creativity within certain bounds of variability. To know is to know the rule-for-making something, which is the procedure that is behind what we call *awareness of kind* or *subsumption under a universal*.

THE INTERDEPENDENCE OF SPACE AND TIME

Time is a condition of apprehending space

Kant makes time (inner sense) and not space (outer sense) the fundamental container of all plurality. This is first of all because a spatial cognition can occur in a moment. spatial cognitions time contains space transcends and includes space—a momentary cognition of three-dimensional space presents only a “slice” of time. Moreover, space can only be apprehended over time. I schematize *space* by drawing a line, thereby combining points into a synopsis that I *take* to be simultaneous but which is actually posited sequentially. I draw a line over time; but by attending to my self-identity across space in this act, I bring this plurality of points into a cognition of spatial unity. Different positions are not *completely* different: they all share my “attendance.” This is how I cognize a unity that *spans across space*. This spanning is originally active—the

activity of apprehension (schematized by line-drawing), but is thereafter thought of as a static unity. Spatial positions remain fixed, but space itself arises as the unity of a trace that was produced sequentially, in a time-taking way. Making space thus depends on time.

Making space takes time, and because space takes time *cumulatively*, space arises as **magnitude**. Magnitude has time as its image, but time has *its* image as space. The two are linked by being generated in the same act. Line-drawing is the primordial act of intuitable world-making.

Space is a condition of presenting time as a plurality-in-unity

Although Kant makes time his fundamental container of sensible plurality, in the B Edition he stresses the priority of space—as condition of presenting objective time order, which is something fixed and thus independent of the series of states that I can take as merely internal. The priority of space is the key element of Kant’s Refutation of Idealism, which was the primary motivation behind the second edition of the *Critique*. Kant defends his epistemology from charges of solipsism by pointing out that a knower can be aware of (read: make intelligible) its own passing inner sense (i.e., as an unitary object of a unitary awareness) only by referring this continual passing to the permanence of space and drawing in it a line, whose inherited permanence *presents* time, inner sense, and therewith the unity of the self.

Time is originally produced through line-drawing. The unity of time could not be intelligible without presenting it as a line, which for Kant means it would have no *concept*. To have meaning is to have a rule, which I must know, and this rule is a rule for producing *instances*.

The particularity of time can only be presented in intuition—and this means space by default. I can only present the plurality of passing intuitions as a unity, I must use space. Only space can allow me to present the absolute difference of passing through the relative difference of position. Space is a plurality-in-unity—a plurality of mere positions.

Kant's ambivalence in the matter of the relative priority of space and time is due to the fact that this priority is dependent on what one is looking at. In terms of *completeness*, time is prior to space, because the former contains the later. But in terms of *intelligibility*, space is prior: I can only show *intuitive* relationships in space.

Space, time, and “this”: schematizing blind synthesis

Space, time, and *this* arise as real (immediate) objects through acts of line-drawing. But why are we not conscious of doing this? We cannot cognize the workings of spontaneous (automatic, unconscious) synthesis because such workings precede the possibility of self-awareness and so cannot be valid objects (and would be part of what Kant demotes by calling *rational psychology*).

The meanings of *space*, *time*, and *this* depend on my ability to do nothing less than carry-out their respective syntheses with conscious intent—i.e., to *schematize* them. I cannot know how a synthesis actually occurs in spontaneity, but if its resultant object *means* anything to me, this fact indicates that I have emulated it and know its rule, which is its semantic content as a one over many instances. So I make the referent (and meaning) of these concepts by schematizing their respective acts of synthesis.

This is how I emulate space, time, and the identity of a reality (*this*) that spans across space and time. The referent of *this* is reality across space and through time—i.e., a *body* and a *substance*.

The construction of this object precedes my later acts of comparison, reflection, and abstraction; and it produces the “transcendental content” that is the content of the abstracted category, just as the empirical image of a cat is the content of the universal *cat*.

Schematism is a condition of assertion and recognition

Schematism is implicit in every act of assertion and recognition: I become aware of any intuition, and thereby conscious of myself as epistemic subject, only by recognizing it as *P*. I say that a grammatical subject (whether intuition or concept) “falls under” a predicate concept because I know how to follow a concept’s to make imaginary instances myself, where the rule is the unity of a kind over the many instances that I can produce by following it. I schematize, in Kant’s example, the universal *dog* by knowing the rule that restricts my act of outline-making to within certain bounds of figurative difference.

The categories have meaning as rules of instance-making

Kant says that I schematize the referents of *this* and *P* through acts of “transcendental time determination.” Since time itself is schematized through line-drawing, I interpret this to mean that the pure concepts are functions of unity that determine the act of line-drawing, thereby specifying it. This would accord with the Aristotle’s original definition of a schema as a specification of a concept, except in this case it is not a concept that is being specified but the constructive activity of line-drawing.

We already know how time as such is schematized: I draw a line while paying attention to a certain feature of my activity. In the case of schematizing time, Kant says that I attend to how my act of drawing emulates the continual passing-away of inner sense. What results is the schematic production of time as line-drawing *taken as an*

emulation of passing. What emulates passing is the successive nature of my act, and what first produces the concept of succession, Kant says, is motion. [B154–55] I emulate, or schematize, passing by moving a point through space (and time).

It is my thesis that something similar must happen for all the other pure concepts. Each determination of inner sense that is necessary for schematizing the referents of *this* and *P* must be a specification of the generic act of line-drawing that originally produces time. I schematize the generic act of synthesis through line-drawing, and I schematize the pure concepts by intending the act of line-drawing in a certain way.

Schemata: empirical (images) vs. pure (activity)

Empirical concepts refer to images, but pure concepts refer to acts. The pure concepts are originally rules or “functions of unity” contained by the three elements of judgment—subject-position, predicate-position, and copula. When the subject- and predicate-positions contain concepts, these functions of unity are the operators of general logic. But when the subject- and predicate-positions refer to sensibility, these functions of unity refer to kinds of necessary connection of point-moments, i.e., the kinds of connection essential to the generic physical object. And in order for these rules to have meaning for me, I must be able to schematize them—i.e., I must be able to carry out their respective combinations consciously. I must draw a line, and attend to some feature of the act of drawing. Each pure concept is a way of intending or interpreting the act of line-drawing so that the act of drawing overcomes a certain *ways of separation*.

But there are other kinds of rules—rules that combine the plurality of passing point-data into the imaginary unity of the generic physical object. These rules are the pure concepts, which are functions of unity in judgment. In ostensive judgment, these rules are rules for combining the a priori ways of separation—**space**, **time**, and **quality**.

Space and time are constructed under the pure concept of extensive magnitude, contained in the subject-position. Quality is constructed under the pure concept of intensive magnitude, contained in the predicate-position. These are called **mathematical** pure concepts, because their referents contain magnitude as their sense.

The physical object is not just sensible content that fills space, time, and quality-continuum. Its contents are also sensible reality *in flux*. Sensible reality *changes*. This, too, if I am aware of it, must be a product of synthesis. And if it has meaning, it must also be schematized. The subject-position contains another function of unity, besides that of extensive magnitude. This is the pure concept of **substance**, which thinks reality as a *real continuant* in time. Here, it is the temporal separation of *reality* that is overcome. The subject-position *this* refers to a property-bearer. What bears properties must be a substratum for their realization—something that can instantiate multiple second-order kinds of properties as well as multiple property-instances within each kind. Only what is spatially extended can bear outer properties, and only what is temporally extended can bear properties that are by definition fleeting. The bearer of properties is the bare particular, and this must be a unity that spans spatial and temporal plurality.

The predicate also contains another function of unity, besides that of intensive magnitude. This is the pure concept of **property**, which thinks reality as a varying (momentary) content, a state of the substance, which properly speaking changes since it perishes. The momentary sensible reality is overcome by a unitary rule that schematizes it continually in a series.

Finally, there is the copula of objective truth, which relates substance and quality. We have seen that both of these are magnitudes. Substance is reality perduring through time, which is the very image of magnitude. Quality is reality as intensive magnitude. But a term that refers to a continuous magnitude is indistinguishable from an algebraic

variable. Now, the relation of subject and predicate is that of condition and conditioned: the subject term (time-value) determines the predicate term (state-value). This is the relation $t \rightarrow f(t)$. Thus Kant conceives the non-analytic relation of truth as an algebraic relation. In other words, Kant reads “This (S) is P” as $t \rightarrow f(t)$.

THE CATEGORIES

Point-moments can only be understood as positions in space and time. But I can also *imagine* them as combined in other ways over and above position in space and time as infinite magnitudes. I do this by drawing a line—this figures their connection in intuition. Kant calls this apprehension. Apprehension is schematized by drawing a line and taking the act as a successive gathering-together of points, in a continual series of moments of positing. This is how I actively and consciously carry out the synthesis in intuition—I move a point. I imagine a continually identical point that moves and occupies different positions, over time. Space and time (as unities) are the result. But I can also imagine my continual positing to instantiate other kinds of combination besides fields of sensible appearance, or direct presentation in intuition. I can imagine my act as instantiating other *kinds* of connection. These are:

Quantity

Under the heading of Quantity, Kant lists three pure concepts: *this*, *some*, and *all*. In general logic these serve to determine the distribution of the predicate over the subject. But in sensible synthesis, and transcendental logic, this form serves a different function. Instead of determining the scope of subsumption, the part/whole relation now applies to a spatial expanse. I make a unitary magnitude (*this*), that I then part in imagination (*some*), and then again recombine into a plurality-in-unity, or totality (*all*). Logical quantification, which in the realm of universals is merely the operating of setting the distribution of the

predicate class over the subject class, is involved in the conception of magnitude—in its rule. The rule is: part a unity and recombine the parts—this, some, all. This act of parting a unity and recombining the parts, Kant says, is a *pure concept*. It has expressions in general logic as traditional logical quantification and in transcendental logic as the parting and recombining of spaces (and also time, since time is the content of space, the content of my act countering the succession of passing-away). I am one actor that can a priori recognize that thinking “S is P,” that is, thinking anything at all, contains an essential parting/combining operation. My unity as a subject makes this possible: many are apprehended, many are parted along the fault-lines of my apprehending, and I am the unity that recombines them into a totality—*all* in general logic, *magnitude* in transcendental logic.

Space

The identity of my act produces space as object. I am one agent that generates space by line-drawing. My identity instantiates an identity across points, so space arises as a whole (after plotting three orthogonal axes). I am the maker of unity, and my unity as a knower is necessary, since a fact is grasped as a kind of unity. I know space as a unity because one actor makes it point-by-point, “little by little.” Its unity is necessary: I cannot imagine it away. Its relations, those of geometry, are invariant—this explains their epistemic status as instances of a priori knowledge. Here, it is my mere identity that I am aware of as I draw, and so the identity of space is taken as empty. (As we will see, when I attend not to my mere numerical identity but to my nature as a power capable of positing images in my own outer sense, just like reality does when I passively receive sense contents, then my spanning across space produces not empty space but material *body*.)

Time

The identity of my act also produces time as object. Passing-away occurs continually. When I move an identical point across space, there is a one-to-one correspondence between a moment of passing and my impulse as power of image-production. If I abstract from my power and consider my mere numerical identity, the result is time as trans-momentary container of moments.

Mathematics

The above remarks explain how Kant conceives the construction of space and time. Space and time arise as formal unities, empty unified frameworks populated with empty point-moments, awaiting stimulation of my passive power. When they are stimulated by noumenal reality, they become filled with a momentary point-content, or elementary appearance. These frameworks are a priori, and explain the possibility of pure mathematics. Because formal space is a priori, pure geometry is possible. Space also provides the enduring required container for adding discrete time-spans by drawing marks, and this makes arithmetic possible. Most importantly, space and time are not only pure frameworks, but also the frameworks of intuition through which I access sensible reality. This gives mathematical cognition objective reality, and so applied mathematics becomes possible.

Physical objects

We have seen that line-drawing produces space and time as magnitudes. In this case, line-drawing is interpreted according to its original presentation in intuition as an extension-in-intuition. The line is here an object that means itself: it is either space, or the time required to draw it. Mathematics deals with mere extension, which is itself an original acquisition, constructed by synthesizing the pure forms of spatial and temporal

separation. Applied mathematics is no different: I am still dealing with the “pure homogeneous”—with spans of space and time as such. When my formal space and time are filled with real sensations, in my acts of mathematical cognition I am still dealing with the underlying homogeneous spans.

But I can connect the points of space and the moments of time in other ways. Kant is looking at *necessary* ways, so these ways must be limited to the “functions of unity” at work in “S is P.” Since we are doing transcendental logic, this “S is P” is facing the passing plurality of outer sense, not the repository of universals which I can then relate in various a priori ways. This “S is P,” which we may call **ostensive judgment**, is an assertion about a sensible fact, which happens to always be a physical object. When I assert truth about sensibility, my topic is a physical entity. All physical entities share certain traits in common. These are naturally called **categories**. The categories arise from pure concepts, which are functions of unity in *judgment generally*, that is, in asserting “S is P” generally. When I assert “S is P” and fill the subject and predicate-positions with universals, I notice certain modifications in my asserting intentionality allow me to predictably determine truth-value. General logic arises from this investigation. But when these necessary functions of unity operate on the plurality of passing point-data, something else happens. Instead of understanding how universals overlap, I cognize a physical object, and I do so as a fact: “This (S) is P.” Kant’s claim is that the physical object arises by imagining the point-data as connected in certain ways. I can imagine these ways consciously, and when I do so I produce their meanings internally. These functions of unity in “This (S) is P” force my imagination to connect point-data in certain general ways, and this produces what we may call the **generic physical object**. These ways of combination are objective: they seem to be in an external object. For this reason, we give them objective names: *magnitude*, *body*, *substance*, *property*, *quality*, and

causality. These are Kant's categories: they are necessary and universal features of all possible physical objects, because they are necessary features of cognizing "This (S) is P" in intuition. The imagination carries out these combinations and presents them as *real*. I do not cognize objects as first-person constructions, but as objects in the third-person. Hence the categories intend third-person entities. But these are combinations *for me*. They are meaningful—and a combination can be meaningful (and not "nothing to me" [B132]) only if I carry it out, or can carry it out. A rule of combination is only good if I can combine. So the meaning of each category must be makeable by me consciously. What are these rules?

We know that the unities are necessary in two ways. First, the rules are necessary functions of unity in judgment, and judgment is necessary for any (epistemic) consciousness. But these combinations also hold their elements together necessarily, because my unity as a knower is necessary. This necessary unity finds its way into the epistemic necessity of the relations that are constructed.

Body

We have already seen the construction of magnitude. Mere identity constructs a space that is a one that can be parted and then recombined, according to the function of logical quantification. But I can also draw a line while attending to something else. I can become aware not only of my numerical identity, but also that I am identical *as a power*. When I posit an imaginary point, I imitate the noumenal reality that stimulates my passive sensibility. This has the form of *change*, and change (Kant says) can only be rendered meaningful, or schematized, by the imaginary act of moving a point.

When I infuse the line with my power, I fill it with a kind of opacity. There is an invisible but real something (impenetrability) filling space. In the *Metaphysical*

Foundations of Natural Science, Kant says: “A **body**, in the physical sense, is a *matter between determinate boundaries* (which therefore has a figure). *The space between these boundaries, considered in accordance with its magnitude*, is the volume [of the body]. The degree of the filling of a space with determinate content is called **density**” [MFNS 64]. This reality is schematized by my act of positing. The physical reality corresponding to it, however, need not contain infinite density: “in the dynamical system of a merely relative impenetrability there is no maximum or minimum of density, and yet every matter, however rarefied, can still be called completely dense, if it fills its space entirely without containing empty interstices, and is thus a *continuum*, not an *interruptum*. In comparison with another matter, however, it is less dense, in the dynamical sense, if it fills its space entirely, but not to the same degree” [MFNS 64]. By “dynamical system” Kant means that section of physics, called *dynamics*, which corresponds to the heading of Quality. This is where Kant treats *reality*, the force of behind sensible stimulation. Reality can be more or less dense across space, *but Kant still treats it as a continuum*.

Including *body* as a category therefore makes sense because it gives symmetry: body is momentary, like property; substance is a continuum, like quality. Again: infusing reality across space yields *body*, and doing so through time yields *substance*.

We will see that time, taken as real object, is likewise constructed by attending to the continuity of my power as agent of positing. (This is dealt with just below.)

Substance

If objective time changed while spanned through it, I would not be spanning through objective time. Instead, objective time would become part of the successive activity of subjective time. Thus: “If we wished to attribute to time itself a succession or sequentiality, then we would have to think yet another time wherein this succession

would be possible” [A183/B226]. The permanence Kant talks about is in fact none other than the stability of spatialized time. When I move about in spatialized time, every time-position is *determinate*. If things moved around, then I would not be aware of the identity of my act, since the very coordinate system through which I was moving would be in a constant flux.

Property and Quality

The value of this principle for mathematical physics (classical mechanics) lies in the fact that I can now put the content (property, in the predicate-position) that I subsume under the predicate and thereby into a relation with time (substance, in the subject-position), which is a magnitude. Variation in state can now correspond to variation of position in the time-continuum. This is the condition of the possibility of mathematical laws of state-change. In other words, the important thing about quality as a value-continuum is the anticipation that *change* will instantiate infinitesimally, that is, *continually*. No new value can leave a gap of mediating difference:

every sensation is capable of diminution, so that it can decrease and thus gradually vanish. Hence between reality contained in appearance, on the one hand, and negation, on the other hand, there is a continuous coherence of many possible intermediate sensations, whose difference from one another is always smaller than the difference between the given sensation and zero, i.e., complete negation. In other words, the real contained in appearance has always a magnitude. [A168/B209–10]

Space, time, and quality are all continuous (or “flowing”) magnitudes. Continuity reigns over space, change in position, time, and change in time-position. And now it also reigns over every quality. A property *could be* any other on the second-order continuum of quality, and if it *does* change to a non-*P*, it will do so in a way that no gap of state-value is presented in the transition between moments. “Hence all appearances as such are continuous magnitudes—both in terms of their intuition, viz., as extensive magnitudes,

and in terms of their mere perception (sensation, and hence reality), viz., as intensive magnitudes” [A170/B212]. Change in quality occurs as movement in a continuum, the very means by which I time and space (and body and substance) are themselves constructed.

Causality

We will now be able to interlink state as property or accident to substance or essence—the invisible reality flowing through time, making time into a real substrate. We will now be able to clock physical objects and relate their state-change to an internal law, “in” the substance. Things have properties that are determined by laws of change. There can now be an a priori science of change, because state is a quantity that is related to another quantity, and quantities can relate through arithmetic as ratios. Ratios, in turn, can hold between entire continuums by means of algebraic relations. The ratio “ $y = 2x$ ” is a relation between two continuums of real numbers. By taking one variable to be *independent* and the other *dependent*, I can determine the value in one variable from the value of the other. This “from” is the grammatical basis of the category of *cause*. Causality is the mathematical predetermination of state as a value of time. This is made possible by the judgment-form of limitation, which schematizes the range of the predicate as a continuum of value-differences, as in our previous *red* example. *Red* means not non-*red*, and both lie on a continuum of *one* way-of-difference, this being the second-order *quality* itself. *Red* is a range within *hue*, which also contains all of non-*red*. Making quality a continuum of real numbers lets me put it into relation with the continuum of real numbers resulting from totalizing time as a series of infinitesimals, or moments. For every moment in objective time, or substance, there is a state-value.

VARIOUS ISSUES RESOLVED BY MY INTERPRETATION

Synthesis becomes intelligible

I take Kant at his word when he says that time is generated, intelligibly for an apperceptive or knowing consciousness, through **line-drawing**. I schematize my unconscious acts of synthesis by interpreting line-drawing according to different rules of “intellectual” synthesis in judgment. This because imagination is productive—it is my capacity to be a force of real sensible stimulation.

Schematism makes knowledge through predication in judgment possible: I know how to use the predicate as a rule for making a matching image myself. Each of the referents of the categories is something that I can recognize under a predicate: *This is a magnitude* or *This is a substance*. These predicates mean something to me, so I must be able to schematize their instances. The something that it recognized by a category is a way of point-moment combination, schematized as a transcendental time determination. So instead of making a content, I make a *kind* of combination. But *any* pure sensible combination is schematized through line-drawing. Once this is recognized, my interpretation of the structure of judgment as an algebraic relation follows automatically.

The interdependence of space and time

Space and time thus arise in dependence on each other, as a hybrid entity—the moving point. We should accept that line-drawing, which is motion apprehended and presented as plurality of positions and moments in unity, is more fundamental than either space or time.

An intelligible theory of transcendental schematism

Pure sensible act: positing a point

The *pure matter* of reality is the posited point-moment. But the *forms* of reality are the ways in which these point-moments stick together in space and time. The first forms are the formal intuitions of space and time themselves as the frameworks for presenting any possible point-moment as situated in a comprehensive field of intuition. But besides the forms of intuition being innate, there are also innate **forms of connection** among point-moments. These, Kant says, are originally the functions of **intellectual synthesis** at work when I intend a true assertion, whose non-optional structure is “This (S) is P.” The subject-position, predicate-position, and copula each contain “functions of unity” that determine how point-moments hang together in the generic physical object. Each such determination is a determination of “**apprehension**,” i.e., the way I imagine point-moments as being intrinsically or necessarily interconnected. This determination is a linking that Kant calls “**reproductive**.” But there is only *one* way for apprehending point-moments, and that is by drawing a line.

Kant says that time is schematized by the act of drawing a line while attending to some *aspect* of the exercise of moving an identical point. As sensible functions of unity, the pure concepts are thus **ways of intending the act of line-drawing**. Just as positing an individual point-moment schematizes the pure content of reality, moving this point in the imagination creates the sensible linking of reproduction that binds the synthesis of apprehension into a “real” or sensibility-pervading unity.

Primary sensible synthesis: moving a point

The essential unity of physical objects, which we have called the unity of the *generic physical object*, is determined by various acts of synthesis carried out by the

imagination. Each aspect of the generic physical object's unity is determined by a certain act of synthesis. And each act of synthesis, in turn, is determined by a certain "function of unity" in judgment. Judgment that aims towards sensibility we have called *ostensive judgment*.

The structure of ostensive judgment is "This (S) is P." Kant holds that the **components of judgment**—the **subject**-position, **predicate**-position, and **copula**—contain functions that determine the unity not only of already-abstracted concepts (such as *bachelor* and *male*) underlying the a priori laws and operations of general logic, but that also determine the act of sensible synthesis. Specifically, since the acts that construct the various unities essential to the generic physical object are sequential and time-taking acts of *apprehension*, Kant calls the components of judgment in their sensible employment "transcendental time determinations." But he says explicitly that time is *originally generated* by drawing a line in a certain way—i.e., by **imagining a moving but numerically identical point**. Doing this makes sensible, or gives objective reality to, the notion of combining real point-moments given in sensibility. I *posit* a point-moment, and thereby act as a force of reality, imitating the realm of reality that sustains the physical world—the **noumenal** force that stimulates my passive power of sensibility.

This act of emulating the object is called **schematism**. It is the way the knowing subject makes an instance of something that it recognizes under a general kind. A **concept** for Kant is a rule for emulating some aspect of experience. I experience sensible contents given by reality in sensibility. To experience something is to recognize it as an instance of a kind. If I experience a green datum epistemically (that is, if I **apperceive** it), I am aware of it being *green*, because I know how to produce all manner of green instances by following a rule that tells me to make a hue bounded by blue-green and yellow-green. But when I experience any point-moment generally, I schematize *it* by

positing a pure point-moment. The fact that the basic element of sensibility is a point-moment is due to something non-optional about my sensibility—something internal to me. These are called the **“forms” of intuition**. But having these forms does not give me knowledge of them. Doing this requires apprehending point-moments into swathes of unified and extended space and time, which are called **“formal” intuitions**.

In the case of schematizing the concept of time (of emulating it by following a rule that constructs it, which is just the meaning of *time* as a concept), Kant says I do so by drawing a line while “attending merely to the act of the manifold’s synthesis” whereby I bring my own inner sense, or awareness of the passing-away of point-moments, into a comprehended unity. [B154] He says I do this by moving an imaginary point, which “taken as the describing of a space, is a pure act of the successive synthesis” [B155 n. 283]. This produces unified or “formal” intuition, which is just a form of intuition (way-of-separation inherent to my very capacity for sensibility) that has been realized in imagination. By positing reality as an identical point and then moving it, I extend my unitary and identical attention across the manifold of sensibility and by doing so assert my own identity as reality-positing force across its dimension of pluralization. The unity of the knowing subject is realized through the unity of an identical act, i.e., the act of positing a moving point and drawing a line. This brings the apperception of positing into an actively apprehending *unity of apperception*.

Form of intuition vs. formal intuition

In an infamous footnote, Kant makes the distinction between two kinds of intuition—form of intuition and formal intuition:

Space, presented as *object* (as we are actually required to present it in geometry), contains more than mere form of intuition; viz., it contains also *combination*, of the manifold given according to form of sensibility, into an *intuitive*

presentation—so that the *form of intuition* gives us merely a manifold, but *formal intuition* gives us unity of presentation. [B160 fn. 305]

The unclear meaning of this has troubled most commentators. But if space and time arise for unitary consciousness as lines drawn by moving a point, then the distinction becomes intelligible. The form of intuition is what belongs to pure sensibility as a way-of-separation. This way is overcome and unified by line-drawing. Line-drawing is the schema of space and time as rules of image-making. I overcome spatial separation by moving the same point to a new position, and I overcome the separation of passing by moving the same point at each passing moment. This is creative apprehension, and it produces a magnitude.

Magnitude clearly cannot be thought prior to synthesis, because it arises by means of combination, and all combination can be meaningful, knowable, and recognized only by being emulated. Line-drawing is the hybrid act that produces space over a duration of time, schematizing space as a line of magnitude taken as a trans-positional plurality-in-unity, and schematizing time as a line of magnitude whose reproduction hangs on my act of moving a point, which emulates passing—the passive content of time. The difference between form of intuition and formal intuition is *magnitude*.

The double function: intellectual rules over sensible synthesis

Kant says that the rules that think the imaginary unities of time are the same rules that I spontaneously think when I intend the **grammatical positions in ostensive judgment**. These rules are originally not semantic contents, but they create instances from which such meanings can be abstracted when they serve as rules of *sensible* synthesis, i.e., as rules of attending to the act of line-drawing, which emulates the basic act of threefold synthesis that is shared by all the particular rules of judgment. There are two logical rules of the subject, and two of the predicate. The subject-position thinks the

unities of *body* and *substance*; the predicate, those of *property* and *quality*. A body is a line taken as a span of reality, objectified as reality in spatial extension. A substance is reality extended through time, which is time itself as real continuant, and real framework of temporal positions. In both cases, the referent is also a magnitude—by virtue of having been constructed through line-drawing. A property is a momentary content of real sensation, just as body is a momentary form of real sensation. A quality is conceived by Kant as a continuum of values, each of which corresponds to a particular content, or momentary property. A quality is a second-order concept. For example, *hue* is for Kant actually a continuum of values, one of which is this particular red content. The final synthesis is the combination of the fundamental container of sensible plurality (time) with the continuum of possible state-value for some chosen quality. Both time and quality are magnitudes, which means that the copula is *a relation between magnitudes*. This relation is determinative (irreversible), and so is analogous to the definition of the subject–predicate relation in general logic, which casts the subject as the *condition* of predication. (The predicate concept *subsumes* the subject concept, but the subject concept has the priority of being the logical condition of truth.) This relation of condition–conditioned, in ostensive judgment, is thus a relation where one magnitude (time–position) determines another (state–value). This is the archetypal and thus metaphysical relation of mathematical physics: $t \rightarrow f(t)$.

In this way, I solve the chief mystery of the Transcendental Analytic, i.e., how it is that “terms” in judgment (**rules of intellectual synthesis**) can perform as rules of objective construction (rules of sensible synthesis), thereby accomplishing Kant’s Copernican revolution which has ontology conform to “logic,” or the conditions of truth. A judgment is true when the predicate is really linked to the subject. In general logic, this link is shared content—“A red triangle is red.” In transcendental logic, this link is the fact

that a given sensible content at time t is supposed to be the value that it is because a physical law has determined this value in a mathematical function taking a time-value as its argument. Ostensive judgment is really an algebraic relation, infused with grammatical intent. Instead of one mere magnitude being related to another as independent variable to dependent variable, this algebraic dependency relation is also infused with a semantic one: the subject variable is time itself, while the predicate variable is a continuum of quality, the universal P that I am predicating of *this, now*. Thus the combination of the schematized referents (two number lines) “in judgment” *really effects* the determinative relation of causal-mathematical unity in nature, as constructed by the mathematical imagination. This brings the combination of the manifold directly into the functions of unity in judgment, and so stands in contrast to Longuenesse’s understanding of the relation between the unity in judgment and the unity in the imaginary object.

Causality belongs to atomic judgment

I bring **causality** back into the fold of a rule necessary for any possible ostensive judgment. One of the root problems for Kant’s Table of Judgments is that, for the pure concepts (forms of judgment) to have a priori objective validity, i.e., universal and necessary applicability, *all* pure concepts must be instantiated in *every* possible physical object. Unfortunately, the only judgment-form that is necessary for all possible judgment is the categorical form, “S is P.” Only atomic judgment is truly necessary, and so only “S is P” (or more properly, for ostensive judgment, “This (S) is P”) is necessarily applied. If a rule of synthesis is necessary, it must be included in atomic judgment alone

The real function of the Schematism chapter

I clarify the function of the **Schematism** chapter. The schemata are the rules for consciously constructing the aspects of the generic physical object that are the images of the categories. Any universal can have meaning only if we know its rule, where knowing is the capacity to consciously construct the instance known. I know that “S is P” when I know that *P* is a rule, and how to use it to make an instance matching the presentation in *S*. The same holds when judgment is ostensive: the sensible instance referred to by *this*. I must still be able, in order to subsume the object under *P*, know the rule that *P* designates, and know that this rule would produce an imitation of *this*.

In the case of empirical concepts *P* that refer to rules of empirical image-making, I learn these through the process of reflection. The semantic content of every universal is determined by the same source—i.e., the content of the instance. But what makes a universal have the “form of generality” is its being a rule for producing like-kind instances, and not itself any instance.

In the case of empirical concepts, I compare particular instances and recognize their differences, and this act allows me to then notice, or *reflect*, what they have in common. I group them, in other words, by dint of properties that are “close” to each other. I am a priori aware of *unified* ways of empirical variation. When I learn a universal, I learn the rule that restricts my imagination to production *of a kind*. For example, hues all have something in common. My power of producing images is restricted, when I know this rule, to variation “within” the class of hues but no other.

The rules in the case of the categories are not learned, but spontaneously followed by imagination whenever I exert the effort towards judgment. The semantic sense of the category rests in the nature of the particular instance that its associated rule, or *schema*, designates. These are the transcendental schemata, which Kant calls “transcendental time

determinations.” To know the meaning of a category is to know its rule, and here the rule is one that synthesizes passing point-data into the unity of the generic physical object. Each of these aspects of the object is a way of unity that Kant identifies with a function of unity in ostensive judgment, “This (S) is P.” To schematize a form of judgment is to consciously emulate the act of combination that this same rule carries out unconsciously during live perception. I have argued that we must extrapolate from the schematism of time, explained clearly in § 24 of the B Deduction, in order to understand the *sense* of the aspects of physical unity that the categories intend.

I have also shown that the synthesis of apperception can be no different from the schematizing acts of line-drawing. The unities ruled by the forms of judgment are necessary for apperception. They are also meaningful for me. Not only can I abstract unclear categories of substance and so on, but I can also make these distinct. When I do, I must rely on line-drawing. This is the only way in which I can emulate the instance of synthesis consciously. This shows that the Schematism chapter is central to Kant’s argument, and not an incongruous addendum (or, worse, a regression to a pre-Copernican notion of category).

This also makes clear that the functions of unity in ostensive judgment are never to be taken as functions on the domain of universals. What are being related are two continuous magnitudes—time and state. The **double function** is not one between rules of universal combination and rules of sensible synthesis, but rules of algebraic combination and rules of sensible synthesis. General logic is a logic of universals and names. Transcendental logic is a logic of interpreted acts of line-drawing that are meant to schematize the meaning of the essential ways-of-combination comprising a physical object, which are a priori mapped onto the components of judgment.

Schematism: decoding the Copernican hypothesis

The real point and role of schematism is to make intelligible our own secret acts of world-making. Kant's own theory itself can only be schematized as analytic geometry: judgment is a relation between variables that relate two real-number continuums.

To schematize a concept is to follow a rule to make an imaginary instance. Kant's thesis is that the physical object is just such an imaginary instance. I schematize empirical concepts by following rules of image-creation limitation: the images "of" that type can only differ so far along certain ways-of-difference. The kind is thus a magnitude of difference along a certain way-of-difference. I schematize any particular property as a value in a continuum of some second-order quality. Thus line-drawing is essential to the function of the predicate: the predicate term itself refers to a continuum of differences. When judgment aims at sensibility, it generates the semantic sense of *kind* by drawing a line as a continuum of qualitative difference. A red instance can only vary along a continuum of difference, and only so far (say, to *orange* and *violet*). And it must do so only within the kind *hue*, which is the higher-order universal that contains it with others of that same higher-order type. Thus subsumption by a universal becomes mathematical in Kant's theory of ostensive judgment. This is the referent of *P*, the pure image that is the referent of the predicate term in judgment.

Meaning for Kant is defined as knowing the rule I must follow when ordering my imagination to produce an instance. This criterion of meaning holds of empirical, mathematical, *and* pure concepts: I must be able to *make* instances of the categories myself, in the imagination. The act of doing so Kant calls *schematism*. Spontaneous synthesis is blind, but can be emulated consciously. Kant calls the schemata of the categories "transcendental time determinations." Since time is originally generated (as an intelligible object) through line-drawing, schematism can only be carried out consciously

as an *inflection* of this act—i.e., by interpreting or intending the act in the way intended by a logical form of judgment, which is its rule. Rule-inflected line-drawing is about “determining” the generation of time in the service of some form of judgment. This is the act that generates the non-empirical unities that constitute the physical object.

The most widely disparaged chapter in the First *Critique* is the chapter on Schematism. For example, W. H. Walsh writes: “The chapter on Schematism probably presents more difficulties to the uncommitted but sympathetic reader than any other part of the *Critique of Pure Reason*. Not only are the details of the argument highly obscure (that, after all, is a common enough experience in reading Kant, though one is not often so baffled as one is here): it is hard to say in plain terms what general point or points Kant is seeking to establish.”¹⁵

One of the key problems is understanding what exactly is being schematized. Some commentators have decided that this must be an already-semantic but “unschematized” category. (See, for example, Werner Pluhar’s translation notes at B159 fn. 298 and A321/B378 fn. 141.) But Kant himself says that, without being schematized, the categories are actually just the forms of judgment as rules of sensible synthesis. This solves the problem of what is being schematized, but it does not explain how it is that forms of judgment *can* serve as rules of sensible synthesis.

Longuenesse sees the Schematism as describing, in specifying detail, the kinds of synthesis that must take place in order for objects amenable to conceptual analysis via “S is P” to become possible: “just those rules of synthesis which provide the discursive forms with the substitutional instances for the ‘x’ of judgment.”¹⁶ But she does not explain these syntheses as *performances* intended through the forms of judgment

¹⁵ W. H. Walsh, “Schematism,” *Kant-Studien* 49 (1958) 63.

¹⁶ Longuenesse, *Capacity* 13.

themselves, but independently of them. Consequently, she never explicitly ties the grammatical positions in judgment to the magnitudes that their respective schemata actually produce. I, on the other hand, will argue that, being in each case an interpreted instance of constructing a continuous *magnitude* by means of line-drawing, the subject- and predicate-positions must be unities-of-thought that are no different from those “thought” under algebraic variables.

In order for synthesis to be intelligible—that is, in order for the rules of synthesis to be *meaningful*—I have to be able to produce instances of these instance-making rules myself. The chapter on Schematism is really about *how* the grammatical elements of ostensive judgment force my imagination to stitch together the pixel arrays in that signature way denoting a physical object. But to make this force intelligible, I have to perform synthesis by drawing a line. In the Schematism, we are explicitly told that each of these aspects of the generic object is a kind of magnitude. To think a magnitude under a grammatical position in judgment can be no different from thinking a magnitude under a term that can be related to another term, and this is no different from the relation of two algebraic variables. I will show that the subject-position, which refers to physical substance, ranges time as a magnitude. Substance, for Kant, is *time itself*, understood as a permanent series of passing states. This is the semantic sense of the term, but the term is also referring to a magnitude. The grammatical position has time-as-substance as its sense, but formally it is a number-line. For this reason, the subject-position *also* functions as an algebraic variable. The predicate-position acts the same way—it is a magnitude, but its sense is a continuum of second-order quality, within which any particular instantiation (say, a particular *hue* or particular intensity of *chocolate*) gains its semantic value. A hue is meaningful only differentially, in relation to *other* hues, and this collection is the universal *hue*, interpreted Kant’s way. Referring to this continuum, which is a magnitude,

is also accomplished by the intentionality of an algebraic variable, again ranging over a number-line. This sets two variables in relation to each other, through the copula “is.” This relation, finally, is causality: the determination of every state (a value) as a function of time (another value). Time and state are set in an irreversible relation. This is the principle of causality explained in the Second Analogy.

Mine is the only theory that shows how Kantian causality has an explicitly mathematical structure. Physics is the science of rendering change as a mathematical predication. Most commentators agree that the goal of the Transcendental Analytic is to shore-up Newtonian science against the corrosive implications of the Humean treatment of representationalism. My model ties this goal directly into Kant’s central thesis, i.e., that the forms of ostensive judgment are rules that unify space and time into a cognition of physical objects—physical objects that are a priori amenable to mathematical reconstruction in our own productive imaginations as objects that arise as mathematically constituted, a priori.

Chapter 1: what is Kant's problem?

KANT'S GOAL (1): DEFEND NECESSARY SPATIOTEMPORAL CONNECTION

The *Critique of Pure Reason* presents two epistemological programs, negative and positive. The goal of the negative program is to determine the bounds of valid knowledge. Humans have valid knowledge of necessary relations only when these relations are logical, spatial, temporal, or deal with certain features of physical objects and the way they change in space and time. Knowledge of necessary relations outside these domains is invalid. Unfortunately, humans are naturally predisposed, by the innate make-up of their understanding, to have certain beliefs that they cannot possibly justify. Kant traces these kinds of unjustifiable knowledge to their source, explains how they arise, and shows why truth claims about them must fail.

The goal of Kant's positive program is to (1) defend the reality of the referents of certain concepts essential to our understanding of physical objects, (2) justify the necessary truth of universally held beliefs about physical reality that are implied by the way we use these concepts, and thereby (3) certify the validity of mathematical physics as a science that determines the state of a physical object as a mathematical function of time.

We all believe that physical objects are spatially extended bodies and temporally perduring substances. We believe that these objects have properties that are momentary and whose reality is continually passing away. We believe that a quality contains a continuous spectrum of values, and that if it changes it must do so in infinitesimal increments over time. And we believe that every change is caused by a previous state of affairs, by which we mean that its occurrence is necessitated by a rule. Finally, we know that mathematics has objective reality—that space, time, and therewith all primary qualities are instances of magnitude.

These beliefs all have something in common. They are all beliefs in certain kinds of necessary spatiotemporal connection among appearances. We think these kinds of connection under certain concepts—*magnitude*, *substance*, *property*, *quality*, and *causality*. These are the Kantian **categories**.¹⁷

The categories are universals and have semantic value, but what they refer to are actually just kinds of necessary spatiotemporal unity—i.e., just those kinds of unity that I recognize as essential features of physical objects in general. For example, when I apply the word *substance* to the world, I intend to refer to something that has no empirical qualities of its own, but which supports them and which itself perdures through time. This, Kant says, is an imaginary entity that is not a reproduction of empirical contents, but is originally produced by the imagining subject. The entity here, Kant says, is not an image, but a *way-of-combining*. I “think” point-data as being stuck together in certain ways by necessity. This sticking is something I have carried out myself, because to understand a combination is to carry it out oneself.

What I intend to refer to when I cognize a substance is something that *perdures through time* despite the fact that intuition only provides me with a passing plurality of appearances. That is to say, to apply *substance* to the influx of passing appearances is to think them as inhering in something that does *not* arise and pass away, but which acts as a qualityless substrate “in which” different appearances are arranged over time. And so, even though I *see* wood or wax disappear in the process of combustion, I *think* that this depiction is in error, and that none of the underlying “substance” has been destroyed. This thought, Kant points out, is functionally identical to the thought that these

¹⁷ While Kant lists twelve categories in the Metaphysical Deduction [A80/B106] and eight in the Schematism chapter [A142–45/B182–84], I list only five. This will be explained in Chapter 5.

appearances are connected to each other across space and through time in *certain necessary ways*.

The kinds of necessary unity referred to by the categories are intelligible and can be articulated as **principles** that express necessary truths about the sense realm. In the case of substance, this is the “PRINCIPLE OF PERMANENCE—All appearances contain the permanent (i.e., substance) as the object itself, and the mutable as its mere determination, i.e., as a way in which the object exists” [A182]. A category is a way of thinking and imagining a necessary connection [A245]; a principle *expresses* this connection as a necessary truth about the sensible world.

By defending the a priori objective reference of the categories, Kant will simultaneously justify our conviction that physical nature is mathematically lawful. When I apply a category, I also assert the truth of a principle that expresses the necessary relation contained in that category’s referent. While this relation has a semantic sense and is referred to by a name, such as *quality* or *substance*, we will see that the referents of these terms really have a mathematical structure. Moreover, and more interestingly, we will also see that these structures are necessary conditions for the practice of mathematical physics.

KANT’S GOAL (2): EXPLAINING THE A PRIORI OBJECTIVE REALITY OF THE CATEGORIES

Showing how it is that the kinds of spatiotemporal connection referred to by the categories can have a priori objective reality is the central task of Kant’s positive program, and the original version of his problem.

- **Kant’s Problem (version 1):** How, under representationalism, can the categories of magnitude, substance, property, quality, and causality have objective reality?

What are the categories?

Inaugural dissertation (1770)

How is it possible to know necessary truths that are applicable *to* the sense-world but not *dependent* on it? This is the question Kant asked in his *Inaugural Dissertation* of 1770, *The Form and Principles of the Sensible and Intelligible Worlds*. By Kant's own later estimation, the answer he gave only adequately explained the objective reality of logical and mathematical truths. The truths of logic are objectively real because they are innate laws of thought, what Kant calls *rules of pure understanding*. The truths of mathematics are objectively real because mathematical objects are made out of pure space and time, which are the forms of pure sensible intuition. The forms in which I construct mathematical objects are the same forms in which pluralize (and interrelate) my immediate awareness of real particulars, so that reality (the world of given appearances) always comes pre-situated in a grid having spatial and temporal magnitude. If space and time were not innate, then the fact that our internal geometrical calculations always have objective reality would be unintelligible. Mathematical truths can be true *always* only if space and time are innate, since we are talking about things as yet unwitnessed.

Making mathematical truths (1) dependent on innate forms of intuition that are (2) themselves conditions of sensible intuition is how Kant explains how mathematical knowledge can be both (1) a priori and (2) objectively real. As a consequence of this explanation, however, the objective reality of my a priori mathematical knowledge is merely phenomenal. Space and time are innate forms of *intuition*—ways in which my sensible states are always organized, not ways in which states of the extra-mental (noumenal) reality are organized.

What about this noumenal reality that produces the changes in my internal states? In the *Inaugural Dissertation* (that is, in Kant's so-called "pre-critical period"), knowledge of real objects is still possible, for the understanding has two uses—logical and real. The logical use of the understanding compares empirical data, abstracts universals, connects these in judgments, carries out syllogistic inference, and subordinates universals hierarchically—all by means of the empty principle of non-contradiction. The real use of the understanding employs pure concepts that give us direct intellectual access, through the very act of *thinking* these concepts, to noumenal objects as they really are. Examples of such pure concepts include "possibility, existence, necessity, substance, cause, etc., with their opposites and correlates" [*ID* § 8]. These concepts are not derived from sensations, but are internal to the faculty of understanding: "Such concepts both of objects and relations are given by the very nature of the intellect, are not abstracted from any use of the senses, and do not contain any form of sensuous knowledge as such" [*ID* § 6].

Thus we have two faculties of knowledge—sensibility and understanding. These provide access to two "worlds"—a **sensible world**, whose objects are phenomena that we passively receive in our sensibility, and an **intelligible world**, whose objects are noumena that we actively think in our understanding. Sensibility represents things "as they appear," while understanding presents things "as they are."

What accounts for the difference between the two worlds is the nature of the faculties that discern them. Sensibility is passive—it receives and re-presents the real object as states of the subject. Understanding is active. In its logical use, the understanding acts by relating and subordinating given concepts. In its real use, the understanding acts by thinking pure concepts that already participate the noumenal being of their intelligible objects. This, in fact, was Kant's original conception of a pure

concept—a concept that gives direct access to reality by being both a meaning for the knowing subject and also a really existing thing, like a Platonic Form. Such a concept is a rule of thinking in the subject and also a rule of reality in the object. A subject who could think beings directly in this way would be capable of what Kant calls **intellectual intuition**. Kant thinks that God is an example of such a subject. [B309]

Letter to Herz (1772)

Kant was dissatisfied with his treatment of the understanding in the *Dissertation*, which “explains” the pure concepts merely negatively by saying that they are not abstracted from sensible representations. In his famous letter to Marcus Herz of February 21, 1772, Kant wonders how spontaneous products of the understanding can have objective reference to objects that, being real, are by definition mind-independent:

our understanding, through its representations, is neither the cause of the object ..., nor is the object the cause of our intellectual representations in the real sense (*in sensu reali*). Therefore the pure concepts of the understanding must not be abstracted from sense perceptions, nor must they express the reception of representations through the senses; but though they must have their origin in the nature of the soul, they are neither caused by the object nor do they bring the object itself into being. (Kant, *Correspondence* 133)

Here, Kant considers and rejects two convenient solutions to the problem of a priori objective reference. The first is that the pure concepts produce the object, in the way that God’s act of thinking is supposed to generate a noumenal object. This is how Kant understands **intellectual intuition**, i.e., the ability to think an object in a kind of immediate, productive relation merely by thinking its concept. The second rejected solution is that the pure concepts are merely abstracted from a physical object whose unity is *given*. Kant rejects both of these options.

The fact that Kant rejects these possible solutions clarifies Kant’s long-evolving problem, which may be stated as: How can concepts that neither generate nor are

generated by their objects have a priori application to an objective reality that is by definition mind-independent? This is the question that the *First Critique*, especially the Transcendental Deduction, is meant to answer.

The clue to answering this question can actually be found lying dormant in the *Dissertation*: the pure concepts arise from “the very nature of pure intellect; not as *connate* notions, but as abstracted from laws whose seat is in the mind, by attending to the actions of the mind on the occasion of experience, and hence as *acquired*” [ID § 8]. The pure concepts arise from the intellect, not as already-semantic universals, but as “laws” of “the actions of the mind on the occasion of experience, and hence as *acquired*.”

The solution given in the *First Critique* is one that both preserves the nature of reality as given *and* makes good on a priori objective reference. Kant must preserve the nature of appearances as reality-produced, i.e., as *given*. If the pure concepts *created* the object entirely, this reality-component would be annulled. But Kant can only preserve the apriority of the pure concepts if the object is *in some sense* dependent on them. Kant solves this problem by making use of the form/matter distinction. The matter of the object is given by reality (albeit in the form of a subjective appearance), while the form of the object is supplied by the pure concepts. The pure concepts are not semantic universals, but *rules for combining point-data*. Only after sufficient experience has allowed us to cognize physical objects can we generate semantic categories, through the traditional **process of reflection**. Kant’s solution, then, is one of *formal constructivism* by means of innate laws of understanding, combined with the ordinary process of concept generation through the process of reflection.

A note on the process of reflection

The generation of universals

In the First *Critique* Kant calls the process of reflection *analysis* or *resolution*. The process of reflection is described in detail by Kant in the *Jäsche Logic*, in a section entitled “Logical Origin of Concepts”:

The origin of concepts as to *mere form* [of generality] rests on reflection and abstraction from the difference of things that are designated by a certain presentation. And here the question arises: *Which acts of the understanding make up a concept*, or—which is the same—*which do belong to the generation of a concept from given presentations?*

Note 1. Since general logic abstracts from all content of the cognition through concepts of from all matter of thinking, it can ponder the concept only in regard to *its form*, that is, subjectively only; not how, through a characteristic, it determines an object, but only how it can be referred to several objects. Thus it is not for general logic to investigate the *source* of concepts, not how concepts *as* presentations arise, but solely how *given presentations become concepts in thinking*—whatever these concepts may contain, something taken from experience, or something thought out, or something gathered from the nature of the understanding. This *logical* origin of concepts—the origin as to their mere form—consists in reflection, whereby arises a presentation common to several objects (*conceptus communis*) as the form required for the power of judgment. In logic, *merely the difference of reflection* in the concept is considered.

Note 2. The origin of concepts in respect of their *matter*, which makes a concept either *empirical*, or *constructed*, or intellectual, is pondered in metaphysics. [JL § 5]

The question that Kant is answering here is *whence universals*: “*Which acts of the understanding make up a concept*, or—which is the same—*which do belong to the generation of a concept from given presentations?*” He is not asking about the origin of the *sense* of the universal. The sense is the particular given in intuition: “In every concept there is to be distinguished matter and form. The matter of concepts is the *object*; their form is *generality*” [JL § 2]. Rather, he is only asking about the origin of their nature as universals.

As Kant says in the Amphiboly, general logic takes all presentations as givens and ignores the issue of their generation. The concepts *triangle*, *red*, and *substance* all have different origins—*triangle* refers to an arbitrary construction in the imagination, *red* refers to a given sense content, and *substance* refers to an imageless combination that we automatically subsume under the subject-position in judgment. In general logic, however, all of this is ignored and each is treated the same way—as a universal. All that is considered are the “partial concepts” that the universal *contains*. [JL § 7] General logic considers presentations as *logical combinations* of partial concepts.

General logic explains the origin of concepts *only regarding their form*—i.e., of generality. This occurs through comparison, reflection, and abstraction. Comparison notices the differences between objects; reflection, their similarities (the genera under which these differences fall); abstraction then isolates these genera and extracts them as concepts.

General logic ponders the concept “not how, through a characteristic, it determines an object, but only how it can be referred to several objects” [JL § 5]. That is, general logic knows nothing about Kant’s claim that judgment-forms can serve as rules that guide the spatiotemporal combination of point-moments into cognition of physical nature, in an act that he calls **transcendental synthesis**. The universal contains marks, but does not relate these to their origin.

For example, the concept *triangle* is originally constructed. But when I consider multiple particular triangles in logical reflection, I only consider their intrinsic properties, which I extract through analytic predication. That is, by means of logical reflection I determine what are the marks common to all triangles.

This explains the “*logical origin*” of the concept *triangle*. What is being generated, or explained, is not the sense (intension) of *triangle*, but only its generality

(extension). The generality of triangle rests on comparing multiple particular triangles, reflecting on their similarities, and then abstracting these from the differences—at which point these similarities become bona fide concepts.

Because general logic generates distinct concepts from given presentations by means of analytic predication, it is capable of producing nominal definitions—even of mathematical concepts. It is for this reason that mathematical concepts, which are for Kant the archetypes of constructive or real definition, can serve as the subject of analytic judgments. Under general logic, *All triangles have three sides* is on a par with *All bachelors are unmarried*.

Thus Kant’s special concepts—the ones that originally construct the object we cognize through intuition—have a double life. On one hand, these concepts function as ordinary universals containing intrinsic marks. On the other hand, these objects are themselves produced by an intellectual act of the knowing subject—the subject *makes* the objects these terms refer to. Thus Kant says that every concept is both a universal and also a *rule for producing imaginary instances*, called a **schema**.

The categories are not rules of synthesis (or schematism)

I mention this here because this clears-up two common pseudo-problems that Kant’s theory is not actually designed to solve. The first is the “problem” of how the categories can serve as rules of synthesis. The answer is that they do not, and are not claimed to. The second is how the categories can serve as the bases of schematism, or the self-willed emulation of synthesis performed by the subject independently of its contact with reality. They also do not serve in this function. The categories are the end products of synthesis, and are only generated, through the process of reflection, from the products

of sensible synthesis guided by the pure concepts, which are originally functions of unity in judgment.

There are only three innate elements in Kant's system—media, rules, and activity. The innate **media**, which Kant calls the “mere *forms* of intuition,” are the ways in which sensibility is pluralized by our power of intuition. These forms mean nothing to the subject, however, until they are combined into the “*formal* intuitions” of space and time, which are meaningful images. The innate **rules** are the functions of unity in judgment, or “judgment-forms.” These guide the synthesis of point-data into the ways-of-combination that are *eventually* abstracted as semantic categories through the process of reflection. The innate activities include the subject's ability to posit images, make rules for positing images, compare physical objects, abstract universals, combine concepts into non-ostensive judgment, synthesize point-moments into the generic physical object, and think the syntheses comprising this object as combined in *ostensive* judgment.

The categories are “original acquisitions”

The process of reflection is one of the central concepts in Longuenesse's treatment of Kant's claim that the rules of physical objectivity arise from judgment-forms.¹⁸ Longuenesse rightly points out that for Kant, neither the categories nor formal intuition (space and time as meaningful objects) are innate, despite their status as a priori. Instead, they are “original acquisitions,” presentations that are *based* on innate elements but products of additional activity:

¹⁸ See, for example, A147/B186: The concepts of understanding do in fact retain a signification, even after their separation from all sensible conditions. But this is *only a logical signification*, [where the concepts of understanding signify] the mere unity of presentations. But these concepts are then *given no object*, and hence also no signification that could yield a concept of the object. Thus, e.g., [the concept of] substance, if one omitted from it the sensible determination of permanence, would *signify nothing more than something that can be thought as a subject.*”

in the case of the categories what is “original” is the *discursive* (intellectual, spontaneous) capacity, with its logical forms as forms of the objective unity of apperception. What is “acquired” are the categories as “concepts of an object, insofar as its intuition is considered as *determined* with respect to the logical functions of judgment.” In other words, what is acquired are categories as concepts of the *unity of synthesis* achieved with a view to analysis according to the logical functions of judgment. (Longuenesse, *Capacity* 252)

Longuenesse is referring to a work that Kant wrote in response to his critic, Johann August Eberhard (1739–1809), who founded and edited a magazine for the sole purpose of attacking the Kantian philosophy from a Leibnizian standpoint. The working hypothesis of these attacks was that whatever is true in First *Critique* has already been said by Leibniz, and whatever is false is the result of departing from Leibniz. In response to these attacks, Kant wrote a paper in 1790 entitled, “On a Discovery According to which Any New Critique of Pure Reason Has Been Made Superfluous by an Earlier One.” The relevant passage:

Only this first formal ground, e.g., the possibility of a representation of space, is *innate*, not the spatial representation itself. For impressions are always required in order first to enable the cognitive powers to represent an object (which is always its own act). Thus the formal intuition which is called space emerges as an originally acquired representation (the form of outer objects in general) ... the acquisition of which long precedes determinate *concepts* of things that are in accordance with this form. The acquisition of these concepts is an *acquisitio derivativa*, as it already presupposes universal transcendental concepts of the understanding. These likewise are acquired and not innate, but their acquisition, like that of space, is *originaria* and presupposes nothing innate except the subjective conditions of the spontaneity of thought (in accordance with the unity of apperception). (Kant, *Correspondence* 136)

So the categories are not innate but “originally acquired.” What *is* innate are the rules of sensible synthesis which are not universals, but pre-semantic “functions of unity” in judgment, rules of “intellectual synthesis.” Thus the categories are based on innate rules, but these rules must first weave themselves into space and time and then be compared/reflected/abstracted as semantic universals. The categories are produced in the

same way as are empirical concepts—they are abstracted or “reflected” from the instances that contain their sense. The pure concepts (judgment-forms) are *original*, but the categories are *acquired*.

The categories are derived *from* experience and do not determine it. However, the categories refer to kinds of spatiotemporal combination that are produced by certain judgment-forms. The categories *name* schemata (judgment-forms geared for sensible employment) and are thus associated with them; but they are not themselves mere thought-forms, but semantic universals.

The second pseudo-problem is what some commentators take to be the problem solved in the chapter on Schematism (which I will treat in Chapter 5). What the Schematism does *not* describe is the conversion of “unschematized” categories, ones that apply to “intuition as such,” into the transcendental schemata that are the rules by which the subject consciously emulates the “blind” act of synthesis, which is spontaneous and automatic. Rather, the schemata are procedures for converting the judgment-forms.

Both of these pseudo-problems are really the same. They are the result of ignoring Kant’s claim that the process of reflection stands at the basis of the generation of the categories. The categories are generated as the *final* step in Kant’s constructive theory of knowledge. They are not innate, but acquired, through the process of reflection.

The hallmark of Kant’s brand of rationalism is its *material emptiness*. There are no innate semantic universals. If there were, the **challenge of representationalism** (and therewith Humean skepticism about the possibility of necessary objective relations) would remain unsolved. The challenge of representationalism, discussed just below, is the problem of how passive subjects that can only access their own internal states can claim to know reality.

Kant's solution to this problem is to grant all the essential claims of British Empiricism but to restrict them to the objects of sensibility alone—i.e., to the domain of passing point-data, before they have been combined by the pure concepts as rules of sensible synthesis. These are rules of combination, not rules of empirical content-creation, and are thus *open* to reality. They do not impose an internal content over the data of reality, which is given, but only add to the relations among these data. Kant's pure concepts apply, as it were, *between* given data. This establishes presentations that are not contents, but ways in which point-moments stick together, necessarily. If categories (universals) rather than judgment-forms were innate, there would be no way for pure (innate) concepts to find their way into *real objects of sensation*. The concept is prior to the *form* of the object, but not prior to its content. This is how Kant's transcendental idealism is a merely *formal* idealism, one whose matter is perpetually open to empirical reality.

THE CHALLENGE OF REPRESENTATIONALISM

The problematic reality of sense contents

A defense of the a priori objective reference of the categories (the physical concepts of *magnitude*, *substance*, *property*, *quality*, and *causality*) is necessary because of the threat posed by representationalism. Representationalism with respect to perception is a kind of *indirect realism*. It is the view that we are never directly aware of physical objects, but rather we are only indirectly aware of them, via a direct awareness of an intermediary mental object. Empiricism is a brand of representationalism. Under empiricism, the only objects of which the knowing subject is *directly* aware are its own internal sensory states. These states are modifications of the subject, not extra-mental or “real” things.

Hume's problematic reality

For Hume, the world of particulars that I directly know is comprised of my own sensations; the sensible world is literally a world made of sense data. Sensations are the only objects with which I am immediately acquainted, so the only “reality” that I directly know is the domain comprised of states of consciousness. And because these states are only momentarily present, the **objective reality** of my knowledge of the sensible world is limited to the collection of content-data I am given in the present moment. Under representationalism, objects *for* consciousness are nothing but momentary sensory states *of* consciousness.

Kant's noumenal reality

The fact that sensations are *given*, that I *receive* them *passively*, indicates the existence of a reality *outside* the realm of sensation. For Kant, sense data therefore have do have a positive relation to reality—i.e., the realm of noumena. Sensations indicate reality because their occurrence depends on something extra-volitional. Sense data are passively produced *in* consciousness by something *outside* of consciousness. This reality is a posit; it is the inferred source of the stimulation of my sensibility. And while I cannot know anything about the *nature* of the entity that generates these data, I do know that it *exists*, because extra-volitional stimulation is a fact.¹⁹

What is ultimately real, then, is the force that generates appearances by stimulating our **outer sense**—Kant's term for our sensible interface with otherness (“outer” here means extra-subjective). Just as consciousness has direct *epistemic* access

¹⁹ For Hume, however, we cannot infer any reality beyond or behind a sensation. The belief that a sensation indicates reality is *nothing but* the “firmness, or solidity, or force, or vivacity, with which the mind reflects upon it, and is assured of its present existence” [T 1.3.8].

only to the *effects* of sensation, outer sense has direct *reactive* access to an extra-subjective sense-stimulator, which Kant calls **noumenal reality**.

Kant redefines reality as appearance-generator

Because outer sense is directly related to noumenal reality, the mere fact that sensation occurs is itself warrant for relating appearances *as contents* to a real ground. So while for Hume representationalism problematizes the notion of reality, for Kant it is an opportunity for redefinition: reality is simply the whatever-it-is that functions as an appearance generator for a subject whose awareness of particulars occurs passively, i.e., extra-volitionally. The important positive upshot here is that, for Kant, the reality of appearances *as contents* is not problematic. Ultimate reality is noumenal reality, while reality-for-us is the appearance as content.

The problematic reality of the *spatiotemporal connections* of sense contents

As Berkeley (and later Hume) pointed out, representationalism problematizes not only our knowledge of the contents of sensation, but also the knowledge of their interrelations. For if I restrict the bounds of real knowledge to what is empirically known, so that I view experience as if it were composed entirely of sensations, then I can only have warranted knowledge about (1) these data as *contents*, and (2) the spatial arrangement in which they happen to be given at a particular moment.²⁰

Example: body as rule-necessitated spatial contiguity

Take, for example, my perception of a solid red triangle. What (under empiricism) do I really know in such an experience? Well, I can know that before me are some red pixels, that these are continually arising and passing away, and they are arranged in a

²⁰ Kant disagrees. For Hume, spatial arrangements are given. For Kant, *space itself* is constructed. Kant deepens Humean atomism, bringing it to the level even of space. This is the key to Kant's solution to Hume's skeptical doubt, as I will show.

triangular cluster. But I cannot know if this unity in the triangular pattern is *real*. I can say: *Here are red data in a triangular cluster*, but I cannot say *Here's a real red triangle*, because this means something else. It means that the triangular arrangement of point-data is not accidental but necessitated by the reality that *produces* these data—i.e., necessitated by a rule inherent to reality as an extra-sensory substrate. To say this with *justification* would require knowing that the triangular arrangement and contiguity of the data has been determined prior to their appearance in me by the force that effects this appearance. But the force that effects appearing is noumenal reality, so this is impossible.

Example: substance as rule-necessitated temporal contiguity

We can appreciate the problem of justifying our knowledge of temporal unity more clearly with another example. Take my perception of a rotating cube. Empirically speaking, all that I know or experience is again momentary: *this present array of point-data*. Each moment presents a different array. What remains constant is only the transparent grid into which each unique array is arranged. Given a series of such arrays, I can justifiably say only that the point data are arranged into one, two, or three quadrangle-like clusters, and that successive arrays give the *illusion* that the quadrangles in one array are ancestors of those in the next. This succession *seems* to denote an encounter with a rotating cube, but that is not what I justifiably know under empiricism. Empirically speaking, all that I really know when I seem to perceive a rotating cube is (1) that some of the data are clustered into quadrilaterals, (2) that these quadrangle clusters are adjacent in such a way that they appear to present one point-of-view of a six-sided three-dimensional object, and (3) that successive quadrangles appear to change their shape and size in *certain regular* ways—i.e., ways that indicate the presence of what appears to be a perduring, physical cube that is rotating. In other words, all that I really

perceive are arrays of point-data that are blinking into and out of presence in a way that conforms to what I would expect from points that were attached to the surface of a rotating cube. By itself, sensibility delivers only a passing plurality of point-data. This is analogous to what is actually presented on a computer screen: **passing pixel arrays**. A computer screen is a grid of empty point-positions that are continually updated with new contents, i.e., color-values such as *hue*, *brightness*, and *saturation*.

A sequence of *regulated* pixel arrays on a computer screen produces the illusion of an encounter with a space- and time-binding force—one that connects the points in one array to each other through their being “attached to” a unified (space-spanning) *body*, and across time with themselves-in-the-future by being attached to a body that is also a unified (time-spanning) *substance*. In the case of an animation of a rotating cube, I am obviously not having a cognitive encounter with a real cube. Rather, the “cube” I perceive is entirely a fiction of my imagination. Nonetheless, the illusion of a real cube is compelling, in that I take the space- and time-binding force to be *just as real as the flashing point-data themselves*. But under empiricism we must limit what counts as real to sense data, and since sense data are subjective states, we would then have to conclude that any assertion involving “this rotating cube” is illegitimate—the result of induction from what is really just a regularity in the passing of arrays. An sequence of passing arrays presents neither a *body* nor a *substance*. In a moment, what is presented is a mere cluster points in space; but not a body, which is a rule-necessitated unity-across-space. Over time, what is presented is only a sequence whose order has only *so far* adhered to the rule-necessitated unity that would belong to a real rotating cube; but not a substance, which is a rule-necessitated unity-through-time.

How can we establish that physical unity is rule-necessitated?

In Kant's model, to *know* that a stable series of triangular arrays of point-data indicates an encounter with a noumenally real triangle (or that a changing series of adjacent quadrangles indicates an encounter with a real cube) would require accessing the extra-subjective sensibility-stimulator that produced them. If there are any noumenal rules by which point-data are spatially arranged, they would have to be laws determining the activity of the stimulator—laws that are *internal* to the stimulator. But the internal logic of the simulator is unknowable. So it would seem that, while the illusion of physical cognition produces an entirely convincing encounter with a real unity (the measurable physical object), and while the necessity of unities that define this encounter is indefeasibly compelling, we could never *establish* that these unities are really necessary. How, then, can we distinguish between connections that are merely rule-like and ones that are “real”—i.e., ones that are rule-necessitated?

KANT'S SOLUTION IN THREE STEPS

Step 1: reality of connections is equivalent to their necessity

Clustering and sequencing is necessitated by rules that are *internal to the subject*. And the rules are necessary in the strongest sense because they are rules that are necessary for bringing point-data into the special kind of unity required for being a knower.²¹ Some ways of spatiotemporal connection really are necessary—necessary for knowing aimed at outer sense. These rules, Kant says, are innate to all humans. This is why we all *necessarily* agree about certain things—i.e., truths of logic, mathematics, and

²¹ Of course, it is not necessary that every physical cognition produce an encounter with a rotating cube. It is not the particulars but underlying “principles” that are necessary. Physical reality is corporeal—made of bits of impenetrable extension. If these bits combine into a rigid cube-shaped body, then the spatial contiguity in any given pixel array (and the temporal histories of the points on its surface) is necessary.

(mathematical) physics. Kant carries out his solution in four steps, which will now be examined.

When we say, within the representationalist paradigm, that a physical object is real, all that we can mean is that the ways in which its constituent data are spatiotemporally interconnected, and the ways in which these data change, are *necessary*—i.e., that there are certain regularities or unities that *must* be encountered, unities from which we *cannot abstract* and still have what would count as a physical object. [A96] This reductive *definition* of real relation with relational necessity is good news for Kant because it opens up a way *within* the representationalist paradigm to establish the reality of physical objects *solely in terms of necessary relation*. Thinking that a physical cube is real is no different from thinking that the imaginary connections among its point-data are *necessary*. Now, it is true that reality *qua* sensibility derives from the extra-volitional ground of the matter of sensation. Every sense-content is an atom of quality and extra-volitional force. But the reality of *connections* cannot be a content. So Kant identifies the “force” of connections with their being necessary. Consequently, to justify my belief in the reality of a physical cube—that its substance is a real “something as such = *x*” perduring through time and that it is only changing its properties (its position in space) by rotating—will require nothing more than establishing that the rules or algorithms which conjure the illusory “rotating cube” (from what might have been taken for a sequence of pixel arrays) remains *constant through time*. Under representationalism, the “reality” of *connections* can be nothing other than their necessity.

So while sense data as contents are necessarily real merely by being presented (since outer presentation is identical with reality), changes-in-presentation are not real by way of presentation. They are real by way of necessity, which *cannot* be presented. To

say that the rules governing the combination of passing pixels into a cube are real is just to say that they are necessary—as necessary as the counter-subjective force that the subject infers from the pixels being produced by the passive faculty of intuition.

But how can we verify that rules determining spatiotemporal connection are necessary? We have already noted that rules grounded in noumenal reality cannot be verified. The noumenal reality that impinges on us and stimulates our sensibility might or might not follow rules. In order to know, we would have to be able to inspect the insides of the stimulator directly. This we cannot do. All we can know about the stimulator are our perturbations, and even these are subjective renditions, not copies.

Like all physicists and most humans, Kant is convinced that connections among point-data really are necessary. His task will be to show, within the framework of representationalism, that certain kinds of spatiotemporal connection are necessary. This is the second version of Kant's problem.

- **Kant's Problem (version 2):** How, under representationalism, can we establish that certain kinds of connection between point-moments are validly necessary?

Since necessary connection requires a ground, and since an extra-subjective ground is unknowable, Kant suggests that it might be worthwhile to attempt grounding this necessity in the knower. This is his famous **Copernican hypothesis**—the notion that structure of the object of knowledge conforms to the structure of the knower. Kant's Copernican strategy follows from the simple fact that the only necessary relations that are verifiable by a knower are ones that are *internal* to it.²²

²² Internal *not* in the sense of being a given object of introspection, but internal in the sense of being produced by the subject's *activity* of knowing. It is my internal act of comparing *bachelor* and *unmarried* (within the internal medium of logical subordination) that ensures the necessary truth of *All bachelors are*

Hume's theory is the culmination of the elimination of realism in the philosophy of perception. Hume shows that the consequence of representationalism (i.e., the ideality of our representations) is skepticism about all non-analytic knowledge. But this result only follows because Hume retains the realist standard of truth and objectivity. Hume was an idealist with respect to our representations, but a realist with respect to the object to which these must correspond. Kant's solution is to make the object-being-represented internal as well. The relation between judgment and extra-mental object is replaced by the relation between judgment and imaginary connections among point-data. Correspondence is guaranteed if Kant can show that these connections have, as their rules, the necessary forms of judgment. The fundamental form of the object is literally the form of judgment.

Objectivity means universal intersubjective agreement, and for Kant intersubjective agreement holds only in three realms—logic, mathematics, and mathematical physics. Under representationalism, the basis of intersubjective agreement can only be *intra*-subjective. Epistemic apriority rests on **genetic apriority**—on innate media, rules, and activity.²³

If Kant can show that the structure of knowledge (specifically, the form of necessary truth) is determined a priori by the media, rules, and activity of the knower, then his Copernican hypothesis will no longer be a hypothesis. He will have shown that the connections among point-data, which we take as referring to physical objects, are forced into cognition by a necessity internal to the knowing subject, i.e., by the necessary conditions of being a knower.

unmarried. It is my internal act of line-drawing (within the internal medium of Euclidean space) that ensures the necessity of geometrical truths.

²³ The original internal media are logical subordination and Euclidean space; time is an acquired medium. Kant calls them “forms,” but the term *media* is more accurate since he views them as fields within which elements can be separated and combined.

Step 2: connections are imaginary

Under representationalism, a physical object is materially constituted out of sense data. And we know that the spatiotemporal connections among these data seem to be (Hume) or really are (Kant) necessarily connected. But what is the nature of these connections?

For both Kant and Hume, the nature of the connections is **imaginary**. The imagination is the ability to produce faint “images” (reproductions of past sense data) in outer sense. For Kant, the transcendental imagination works through the self-stimulation of outer sense²⁴ by reproducing empirical contents and positions them in space. [B154]

For example, in order to cognize a body or other spatial expanse, I must produce it by drawing an imaginary line—by moving a point through a series of positions *continuously*. This produces the contiguity of points necessary for the cognition of a unified spatial expanse. And when I cognize a physical object, I must imagine the sequence of past pixel arrays as being linked to the present one according to some unitary rule (such as the rule “rotating cube”) so as to present the kind of unity that denotes a perspectival history of some identical object. These activities, by which I “think” clusters and sequences as rule-necessitated unities, are examples of **transcendental synthesis**. Transcendental synthesis is Kant’s term for the imaginary construction of the essential aspects of the generic physical object.

I can also produce an imaginary animation that shows *what would happen if* I were to move or rotate the object in a certain way, based on *what happened when* I did so earlier (or based on other clues). For example, when I perceive (thanks to synthesis of

²⁴ Recall that outer sense is the field of simultaneous plurality, while inner sense is the field of passing away. Since I can take outer data as being “mine,” these are also contained by inner sense. For example, my visual perception of a table can be taken as a “time slice” in my experiential history. The reverse does not hold, however

point-data into a unitary body) what looks like a cube from a point-of-view, then my imagination can also produce a simulation of what would happen if I were to move that cube (or my viewing angle) in a certain way. This is called **transcendental schematism**. Cognizing the cube's (partial) appearance in one momentary pixel array *as revealing a physical object* is identical to imagining its necessary connection to the sequence of arrays before and after it.

Imagining a moving point in order to apprehend space, and imagining a motion picture in order to reproduce pixel arrays in time are the object-making acts of imagination. The difference between Kant and Hume lies in whether or not such imaginary connections *can be necessary*. For Hume there are no necessary imaginary connections. Words like “always” and “must” have no valid application to imaginary unities because what drives the imagination to spatiotemporally associate two images is merely the force of habit. For Kant, however, the connections denoting physical reality are just as necessary as those of math and logic. But how can this be established?

Step 3: imagining as essential to knower-making

Kant agrees with Hume that (1) our access to empirical reality is limited to the contingent plurality of momentary sense data and, consequently, that the necessity of extra-subjective connections can never be established. Kant also agrees with Hume that (2) the referents of our concepts of necessary relation (the categories) are imaginary, i.e., produced by the subject. But he does not agree with Hume on an implied third point: (3) that imaginary connections cannot be necessary.

As we will see in Chapter 2, Kant argues that some kinds of imaginary spatiotemporal connection are necessary because they are necessary *for the possibility of knowledge in general*. And by “possibility of knowledge” he really means the possibility

of *knowing*—i.e., the possibility of a unitary judging *knower*. Kant will argue that the subject of knowledge cannot function as a knower until it is *made*. The knower depends on necessary imaginary acts of what we can call **knower-making**.

This unhappy phrase is unfortunately the best way to render the idea behind Kant’s claim that the subject is the final cause of the object’s unity. We cannot use “self-making” since the self for Kant refers to a noumenal reality that we cannot know. And we cannot use “subject-making” since the term *subject* also refers to the subject-position in judgment. Also, there is a subject of sensibility and a subject of understanding, and they are different. Only *knower* expresses what Kant means: that self-awareness arises only as epistemic consciousness, i.e., as a being that asserts “This (S) is P.” Knowing brings *both* subject and object into awareness. Kant’s term for knower is *apperception*. The identification of apperception with knowing will be discussed in detail in our treatment of § 16 from the B-Deduction in Chapter 4.

Kant defends *objectively* necessary relations under representationalism by showing that they are just the objective reflection of the imaginary acts of spatiotemporal combination necessary for knower-making. Necessity *in* some objective representation is justified by interpreting it as necessity *for* any and all objective representation. Under representationalism, knower-making determines world-making. Every sense-consciousness is also a datum, so imaginary connection of sense-consciousnesses into the unity of the knower is also, when taken in the accusative, connection of sense data into the unity of the known.²⁵

Recall the rotating cube. Instead of experiencing a sequence of pixel arrays *as* a sequence of pixel arrays, I experience them as time-slices in the unified history of a

²⁵ This Janus-faced nature of the stimulation-event as both *my* (possessive case sensation) and *that* (accusative case cognition) will be explored in Chapter 2.

rotating cube. The experience of the cube is *nothing but* my spontaneous impulse to imagine the pixels as being connected through space and time according to certain rules—i.e., those comprising “an encounter with a rotating cube.” But these rules in turn rest on deeper ones. I could not conjure the imaginary experience of a rotating cube unless I came prepared with rules for synthesizing (and recognizing) pixel array sequences according to the more general rules comprising “an encounter with a physical object.” For Kant, the rules of physical unity are necessary features of the world because these rules are also (and originally) genetically a priori rules necessary for knower-making.

This is different from Hume’s theory. For Hume, the knowing subject is a ready-made unity, and does not require any preliminary operations of imagination in order to intuit space, cognize time, or think facts (make judgments). For Kant, however, the plurality of sense data is also a plurality of sense-*consciousnesses*—a plurality that must be combined into a unitary **epistemic consciousness**. Sense data are combined in certain necessary ways because sense-*consciousnesses* must be combined in the ways necessary to produce the unity of a viable knower. The ways in which I imagine the sequence of pixel arrays being necessarily connected *are also* the ways in which my productive imagination must combine the plurality of sense-*consciousnesses* into unitary epistemic consciousness. The ways of combination necessary for knower-making are carried out (unconsciously) by the subject and are then encountered as the necessary principles of physical objectivity. Necessary connections in the knowing subject become manifest to that knower as necessary connections in the known object.

Kant locates his ground for epistemic necessity in the construction of the *cogito*. Kant interprets the necessary conditions of knowing as necessary conditions of a *unified knower*, as conditions for the possibility of intending “I know.” The “I know” is Kant’s

constructivist version of the Cartesian “I think.” Kant conceives the genesis of the “I know” as the act of putting the original plurality of sense-consciousnesses together into a unitary epistemic subject. What are the necessary conditions of this unity?

Kant determines the conditions of the “I know” by pre-supposing its *non-existence*. This is Kant’s strong way of deriving necessary conditions—from nothing, from the position of a failed knower. He asks *about* the knowing subject, from the privileged perspective of a transcendental psychologist: *What are the kinds of ways in which the “I know” could fall apart? What are the kinds of togetherness which, lacking, would undo the unity of knowing?* And he asks from within the horizon of the knowing subject: *What are the fundamental (original, genetic) conditions of togetherness that I find in my experience?* The answer is that I am a knower only under certain conditions of unity—i.e., only when my sense contents are combined in imagination across space, time, and in the ways required in order for judgment to be possible.

Kant will argue that certain acts of imagining are essential to the process of knower-making—i.e., the process whereby the knowing subject is first put together. Showing that acts of imaginary combination are necessary to knower-making is the third version of Kant’s problem.

- **Kant’s Problem (version 3):** How, under representationalism, can acts of imaginary combination be necessary to knower-making?

This necessary activity of imagination is transcendental synthesis, and the faculty that carries it out the **productive imagination**. The productive imagination “is a power of determining sensibility a priori; and its synthesis of intuitions *in accordance with the categories* must be the transcendental synthesis of imagination” [B151–52]. The

productive imagination stands in contrast to the Humean (associative) power of imagination, which Kant calls **reproductive imagination**.

Review of these steps

Kant's original goal was to show that the categories have a priori objective reference—that our universally shared concepts of necessary spatiotemporal connection have real referents *in the world*. Kant's approach to solving this problem involved reformulating it in three stages:

1. How can we justify our belief that the essential connections denoting physical objectivity are rule-necessitated instead of merely rule-like? Representationalism prohibits us from appealing to an extra-mental ground because the latter is inaccessible. But there is nothing prohibiting the possibility that these rules are internal and accessible to the subject. Kant's Copernican hypothesis is that the connections denoting physical objectivity are necessitated by rules necessary for the possibility of knowing. Kant substitutes a *pre*-mental ground for the extra-mental one of earlier realistic epistemologies.
2. So representationalism forces Kant to suppose that the rules that necessitate certain kinds of spatiotemporal unity be internal. But it also forces him to suppose (with Hume) that the connections themselves are internal as well. The connections denoting physical objectivity are acts of the *imagination*.
3. But how can imaginary connections be shown to be necessary? Kant's answer is Cartesian, with a twist: they can be shown to be necessary by showing that they are necessary *for the cogito*. To be necessary for the "I think" means to be necessary for *knowing*. Kant takes this necessity in a strong Cartesian sense—of acts of imagining *that are necessary for there being a knower*. These acts are the necessary constructive conditions for the possibility of a unitary epistemic consciousness.

DISCOVERING THE WAYS OF KNOWER-MAKING

What is really innate—media, rules, acts

Kant's decision to approach the necessary conditions of knowledge in terms of knowledge *composition* is his way of implementing innatism in a way that does not rely on innate ideas. There are no innate universals for Kant, only innate **media** (or “forms”) of space and time that permit us to combine data, innate **rules** that force certain kinds of combination rather than others, and innate **acts** that carry out combination. Media, rules, and acts must be innate because they are the conditions of combination, and combination is the “only [presentation] that cannot be given through objects, but ... can be performed only by the subject himself” [B130]. Kant's theory of innate ideas is minimal and formal—what is *innate* is limited to only what is *necessary*, and these are rules of combination.

Pure concepts cannot gain objective reality by positing empirical contents

Under representationalism, it is only in the act of *constituting* something that my knowledge can be perfect. Recall the definition of reality as whatever is extra-volitionally given in intuition. My pure concepts cannot gain objective reality *here*. I cannot posit a sensible content without displacing what is given. However, there is room *between* these contents—i.e., in the undetermined “space” that is their relation or connection. The pure concepts, which are originally functions of unity in judgment, gain objective reality not by replacing point-data, but only by supplying their relations. That is to say, the pure concepts are nothing but rules that necessitate certain ways of thinking the spatiotemporal connection of point-data. How does Kant explain this?

Innate media

Since it is none other than *I* who unify sense data into knowledge (indeed, into the very object whose structure is that of judgment), it must also be *I* who contain the media in which this putting-together occurs. A medium is necessary for any combination—it is the field of awareness within which combination and analysis occur. There can be no combination (or separation) of elements without a **shared dimension difference** in which they can all be interrelated. In fact, it is the medium that determines the *kind* of a combination, and therewith the nature of the combined product.

For humans, these media are **space** and **judgment**. Their corresponding kinds of analysis/synthesis are, respectively, spatial separation-and-combination and the conceptual separation-and-combination carried out in judgment. The medium of space is innate. This has been carried over from the *Dissertation*. The medium of judgment is logical identity and difference and logical subordination.

Innate rules

The innate rules that link-point data are the functions of unity in **ostensive judgment**, i.e., the act of asserting truth about the passing plurality of point-data delivered by sensible intuition. In order to apply judgment to the passing plurality of outer sense, the structural **components of judgment**—the subject, predicate, and copula—must have appropriate referents. It is the demand for these referents by our effort to understand (by means of judgment) that makes the rules that guide their construction mandatory. The rules necessary for thinking the referents necessary for the applicability of judgment to sensibility are Kant's pure concepts.

Innate acts

The third innate Kantian element is the knowing subject's epistemic activity. The subject itself, as spontaneous agent, is the prime mover in this process of referent-construction, which is also the construction of the physical object. The acts of construction are acts of imaginary spatiotemporal combination, which Kant calls transcendental synthesis. The telos that guides this synthesis is the drive to understand—the merely spontaneous subject seeks to become an epistemic subject, a knower, and this requires the activity of imaginary combination.

KANT'S METHOD OF DISCOVERY

Since the ways of knower-making manifest *in* knowledge as necessary truths, we can learn the necessary ways of knower-making from the *kinds of necessary relation* that are contained in whatever bodies of necessary truth we happen to know about. Kant will carry out his method of discovery in the following steps:

1. Kant's system ultimately centers around explaining the imagination's activity of physical world-making. But this activity is occult and cannot be directly accessed. How can Kant discover its forms and rules?
2. The answer is that he will *infer* them by examining bodies of necessary truth that have already been established. For Kant, "necessity and strict universality are safe indicators of a priori cognition, and they do moreover belong together inseparably" [B4]. Since epistemic apriority is the result of genetic apriority, Kant thinks that the former can serve as a guide to the latter.
3. To discover the forms and rules of knower-making, Kant will turn to **geometry** and **logic**. Geometry and logic are sciences whose principles have unquestionable epistemic apriority, and are recognized even by Hume.
4. According to Kant, the reason geometry and logic are a priori is because their *relations of unity* arise with the act of knower-making. The necessary truths of these sciences are expressions of necessary ways of knower-making—i.e., ways

- of separation and combination that, if they did not exist, would make *any* knowing impossible.
5. The ways of combination necessary for the knower are reflected as necessary combination in the object of knowledge.²⁶ When we articulate the necessary truths of geometry and logic we are, Kant says, actually reporting on the ways in which epistemic consciousness has *already* been put together in the act of knower-making. Necessary truth is the objective-factual face of necessary acts of knower-making. Therefore, we can use the structure of necessary truth as a model for the kind of construction involved in knower-making.
 6. Kant's job, therefore, will be to devise a story of knower-making that concludes with the kinds of necessary truth desired. He will tell one story of knower-making that ends in the production of Euclidean space, and another that ends in the production of the system of logical coherence and its a priori rules.
 7. The result of knower-making is unitary epistemic consciousness. Epistemic consciousness has an intelligible structure that it realizes through its own act of self-articulation. This is the **structure of epistemic consciousness**: "I think that this (S) is P."
 8. We will see that geometry and logic correspond to the two main parts of the structure of epistemic consciousness, which we may call *intuiting unity* and *judging unity*. By examining the necessary relations in these sciences, we can determine what kinds of knower-making must be at work in the structure of epistemic consciousness.
 9. The **intuiting unity** of epistemic consciousness arises from the subject's spontaneous act of combining sense-consciousnesses into a unitary intuiting consciousness. This is the unity of the "I" underlying the "I think"—the first component in the structure of epistemic consciousness. Due to our innate **form of sensibility**—and the fact that every sense-consciousness is also a sense datum—the combination of sense-consciousnesses results in the unified **formal intuition** of space. Euclidean space results from the combination of separate sense-consciousnesses into unitary intuition. The reason geometrical truths are necessary is that the knower does not acquire unitary intuiting consciousness until it intuits sense data as arrayed in unitary space.

²⁶ Since every empirical content is both a state *of* consciousness and a potential datum *for* consciousness, a datum can be taken either in the possessive (**my awareness** of the object) or in the accusative (my awareness **of the object**).

10. *The synthesis that carries out this unification is the act of **drawing a line**. The isomorphism between the space of geometry and the unity of intuiting consciousness is perfect. For this reason, the structure of the intuiting self can be read from the structure of geometry. Euclidean space is the framework of intuiting consciousness.*
11. The **judging unity** of epistemic consciousness arises from the subject's spontaneous act of combining concepts in judgment—i.e., in an act of logical subordination that claims truth. This is the unity of the “this (S) is P”—the second component in the structure of epistemic consciousness. This combination is the unity of thinking the S-concept under the P-concept. The reason logical truths are necessary is that the knower does not come together (and understand, through judgment) until its plurality of sense-consciousnesses, already combined into intuiting unity, come under the higher unity which thinks “this (S) is P.”

The isomorphism between the traditional science of logic and the unity of judging consciousness is only partial. The traditional science of logic treats the unity of judgment solely as a relation between *concepts*. Kant calls this traditional approach **general logic**. General logic treats judgment as the subordination of one universal (the subject) under another (the predicate).

But this cannot be the function of judgment when it is applied to the passing plurality of sense data. In the case of applying judgment directly to sensibility, which we have called *ostensive judgment*, the function of judgment is not one of concept subordination, but something much more complex. Thus there is a new logic, one discovered by Kant, that explains how it is that the **structure of judgment**, “S is P,” can be successfully applied to the passing plurality of outer sense. This new logic is Kantian **transcendental logic**.

Chapter 2: Kant's theory of knower-making

A SIMPLIFYING MODEL

The world of physics arises from acts of knower-making that combine the passing plurality of sense-consciousnesses into the unity of epistemic consciousness, whose structure is “I think that this (S) is P.”²⁷ To understand how Kant conceives this combination we need a clear picture of the problem that this combination is designed to overcome—i.e., the problem of the lack of a knower. Here, I will clarify the details of Kant's theory of knower-making by mapping them onto a simplifying model.

Outer sense

Imagine the ontological subject as a sphere surrounded by **noumenal reality**, which is the ontological object. The surface of the sphere is the subject's immediate interface with this reality; through the surface, subject and reality make direct contact. But in this relation the subject is only passive: its surface only registers the ways in which it is stimulated by the power of noumenal reality. This surface is called **outer sense**.²⁸ Through it, the sphere accesses, not the true nature of noumenal reality, but only its own

²⁷ In non-ostensive judgment, which is the kind studied by general logic, the structure of judgment is “S is P,” and admits of certain a priori truth-functional operators: “all,” “some,” “is,” and “is not.” Thus “All S are P” and “Some S are not P” are instances of non-ostensive judgment. But ostensive judgment, or judgment that aims towards sensibility, has as its elementary form “This is P.” It is not necessary to recognize the referent of “this” as an instance of an object-kind, but it is necessary to assert a predicate. This is the simplest type of ostensive judgment, and thus the only essential one. For this reason, I have rendered the subject-position in parentheses. “This is red” and “This rose is red” are both valid ostensive judgments.

²⁸ The “outer” in “outer sense” is only meant to indicate its relation to the extra-subjective, i.e., what is “outside” the subject in the *ontological* sense. Through outer sense the subject relates to otherness. This externality is not, however, spatial. Space is the form built into the *inside* of outer sense. The relation that “outer” sense has to noumenal reality is the source of our sense of extra-subjectivity, alterity, otherness. Noumenal reality is “outer” in a way we cannot understand. But the separation relations that are presented *in* outer sense are spatial. The objective world that the subject cognizes is totally different from the noumenal world in which it is situated. The world that it experiences—its objects, properties, and relations—is an internal formation constructed out of internal states. The same goes for what we can imagine. Inner and outer in our model are ontological indicators; but, building as we are in our own imaginations, we are forced to render these ultimate notions intrasubjectively.

internal states, which are *reactions* to its being stimulated by an unknowable external power.

By stimulating the sphere's surface, the noumenal power perturbs it and so forces a modification of the subject's state. Perturbations begin on the surface of the sphere, propagate inwardly, and are then interconnected according to innate media and rules. The product of this interconnection is **experience**—i.e., cognition of the world of physical objects situated in space and time. The perturbations produced by the stimulation of the sphere's surface ultimately become the “facts” that the subject knows through acts of ostensive judgment.

Sense-consciousnesses are also sense data

Stimulation by outer sense generates consciousness by perturbing it, resulting in the arousal of sense-consciousness. But this same perturbation is also a content, a quality that will eventually be a datum *for* the epistemic subject. Content arises with consciousness—the two come together or not at all. A perturbation thus has a double nature: it is both the arousing of a **consciousness** and the generation of a **content for consciousness**. Kant's word for this original entity, what I have been calling a perturbation, is **presentation** (*Vorstellung*).

The subject is conscious only when its outer sense is perturbed. The very existence of sense-consciousness is dependent on outer stimulation.²⁹ Sense-

²⁹ This is the point of the **Refutation of Idealism**—a special section Kant added to the B Edition of the First *Critique*. Consciousness originally arises by being stimulated into existence by some heteronomous force, i.e., the noumenal reality impinging on outer sense. Since consciousness arises with its stimulation, the very existence of consciousness is *dependent* on outer sense. The dependence of consciousness on extra-subjective reality inverts the primacy of consciousness over matter assumed by Descartes because it shows that the existence of consciousness, contrary to Descartes' presumed order of discovery, “*can be determined only by reference to something linked with my existence that is outside me*” [B xl]. Consciousness as existence depends on the noumenal object that produces it by stimulating *outer* sense. This is Kant's refutation of Cartesian solipsism, the claim that only the mind and its states are knowable.

consciousness is existentially passive—it does not exist except by stimulation of outer sense. When a wave of perturbation rises, so does sense-consciousness; when a wave passes away, sense-consciousness does as well. Because of this, sense-consciousness is not epistemic; that is, it is not awareness of a fact. Fact-awareness requires that consciousness be able to relate to its concomitant content by *transcending it*—spatially, temporally, and conceptually. To be epistemic, consciousness must be able to *intuit* the content as a particular datum in space, *imagine* it in time, and *recognize* it as being of some general kind by subsuming it under a universal in judgment—as in *This is P*, *This is an S*, or *This S is P*.

Kant distinguishes non-epistemic sense-consciousness from epistemic consciousness by calling the former “presentation” and the latter “presentation with consciousness,” or **perception**. Sense-consciousness is pre-epistemic, and thus pre-*conscious* in Kant’s view. Perception is *epistemic* sense-consciousness, or more simply **epistemic consciousness**.

Kant’s originating problem: the unity of epistemic consciousness

The stimulation of outer sense in fact produces a *plurality* of perturbations—and so a plurality of *consciousnesses*. Epistemic consciousness, however, is *axiomatically unitary*—it is one consciousness. For there to be a something known, the plurality of sense-consciousnesses *must* become unified in the ways required by knowing-about-something—into the unity of an epistemic consciousness that is aware of its numerical identity as the subject of knowledge. This is the *absolute* necessity undergirding all other forms of necessity, genetic and epistemic: “The synthetic proposition that all the varied *empirical consciousness* must be combined in one single self-consciousness is the absolutely first and synthetic principle of our thought as such” [A117 fn. 138].

Sense awareness originates as a plurality of sense-consciousnesses, but it is converted into unitary epistemic consciousness. It is this transition from the original plurality of sense-consciousnesses to the self-conscious unity of epistemic consciousness that Kant's theory of knower-making is supposed to explain. This is the fourth version of Kant's problem, which I have called the problem of knower-making:

- **Kant's Problem (version 4):** How, given the original plurality of sense-consciousnesses, can unitary epistemic consciousness arise?

For now, we can trace a preliminary outline of Kant's solution by tracing the steps in his inventory of presentations:

The genus is presentation as such (*repraesentatio*). Under it falls presentation with consciousness (*perceptio*). A *perception* that refers solely to the subject, viz., as the modification of the subject's state, is *sensation* (*sensatio*); an objective perception is *cognition* (*cognitio*). Cognition is either *intuition* or *concept* (*intuitus vel conceptus*). An intuition refers directly to the object and is singular; a concept refers to the object indirectly, by means of a characteristic that may be common to several things. A concept is either an *empirical* or a *pure concept*; and a pure concept, insofar as it has its origin in the understanding (not in the pure image of sensibility), is called *notion*. A concept framed from notions and surpassing the possibility of experience is an *idea*, or concept of reason. [A320/B377]

Presentation: identity of proto-subject and proto-object

A perturbation arises as both content and consciousness. The two are originally indistinguishable. On one hand, the perturbation is an extra-subjectively generated stimulation of outer sense, and in this respect it has the characteristic of otherness. On the other hand, the perturbation exists *in* the subject (it is made *of* mind-stuff) and so has the characteristic of a subject. Kant calls these opposed characteristics of the perturbation, or original presentation, "outer" and "inner."

The **outer** aspect of a perturbation is its aspect of otherness, which derives from its having been produced by extra-volitional stimulation. This provides the matter of what will eventually become the *object*. The material essence of the object is its ontological otherness from the subject. A perturbation is a perturbation of consciousness *by something* else. This is what Kant means by “outer.”

The **inner** aspect of a perturbation is the medium in which it manifests, i.e., the *res cogitans*. A perturbation is a perturbation *made of consciousness*.

These aspects of a perturbation are latent until the perturbation is brought into the unity of epistemic consciousness. When this happens, the duality of outer/inner is transformed into the realization that every datum “refers” simultaneously to both subject and object. When consciousness of a presentation becomes epistemic, it is called **perception** or **appearance**. Taken in reference to the subject, a perception is called **sensation**; in reference to the object, **cognition**.

Perception: epistemic consciousness as subject/object distinction

With perception, the duality of the presentation as inner/outer emerges *into knowledge* as the duality Kant calls sensation/cognition: “A *perception* that refers solely to the subject, viz., as the modification of the subject’s state, is *sensation* (*sensatio*); an objective perception is *cognition* (*cognitio*)” [A320/B377].

How do the latent aspects of inner/outer emerge into knowledge as sensation/cognition? The original aspects of the perturbation are “referred” to the two parts of the **structure of epistemic consciousness**—the “I think” and the “this (S) is P.” The first part intends the subject; the second part, the object. This means that the object is actually an objective fact having the structure of judgment as its form. The subject and object poles arise with the very emergence of epistemic consciousness.

How does this automatic referring of the two aspects of a perturbation to the two poles of knowledge (subject and object) occur? Recall the two aspects of a perturbation—it is made *of* the matter of the subject but *by* the noumenal other.³⁰ These aspects (material/generative) are picked out by the two components of the structure of epistemic consciousness: the “I think” and the “this (S) is P.” So perturbation can be taken *in knowledge* in two ways: (1) as a modification *of* the *res cogitans*, and so as a content made *of* consciousness and referring to the “I think,” and (2) as a fact *for* the subject, a counter-subjective item of which it is (potentially) aware, after asserting its epistemic intent through judgment, which externalizes the perturbation *as* a fact, as the referent of the “this (S) is P.” The “I think” picks out the material aspect and refers it to the subject; the “this (S) is P” picks out the generative aspect and refers it to the object. The former is called **sensation**; the latter, **cognition**.

SENSATION: EMPIRICAL CONTENT ABSTRACTED FROM SPACE

A sensation refers to the subject, but it corresponds to the real. [A175/B217] Sensation indicates the real *as it manifests* in the inner domain of the subject. To refer a perturbation to oneself is to feel it as an immediate affection of one’s internal matter.

But when Kant defines **sensation** as the “matter of perception” [A167/B209], he is abstracting it from the way it actually appears. By calling sensation the “matter” of perception, Kant is treating it in abstraction from the *form* of sensibility. “Whatever in an appearance corresponds to sensation I call its *matter*; but whatever in an appearance brings about the fact that the manifold of the appearance can be ordered in certain relations I call the *form* of appearance” [A20/B34].

³⁰ By “matter” Kant means the passive component of consciousness, the aspect that both (1) *takes on* and embodies the nature of the perturbation provided by outer sense (and thus provides the best indication of the nature of noumenal reality), and (2) acts as the matter that is determined by the norms internal to the subject.

In truth, every sensation is both a content and an outer form—a spatial position, an extensive magnitude, and a figure. We can never cognize mere matter, but we can abstract it from form and, by so doing, arrive at the notion of *mere* secondary quality.

Kant's point in isolating the matter of perception is to isolate the aspect of a perturbation that will eventually be called *intensive magnitude*. He is trying here to isolate the **empirical matter** of a perturbation—not its figure, extension, temporal position or duration, but what will come to be cognized (post-synthesis) as its **secondary quality**. And he will argue that we have a kind of a priori knowledge about this as well, i.e., we know a priori that a secondary quality can be quantified in terms of “intensity.” A sensation has a magnitude of intensity—it can be strong or weak—depending on the intensity of the stimulation of outer sense produced by noumenal reality.³¹

COGNITION (1): THE INTUITING UNITY OF EPISTEMIC CONSCIOUSNESS

A sense-consciousness cannot be taken objectively (i.e., cognized) until it has been combined with all other sense-consciousnesses into the intuiting unity of the “I.” Connecting each sense-consciousness to one and the same “I” is carried out by the productive imagination. Specifically, as we will now see, it is carried out by imagining the motion of an identical point in an act of **line-drawing**.

Form of sensibility: the difference shared among sense-consciousnesses

How can sensations all relate to unitary consciousness *as* sensations, i.e., as different elements of the same *kind*? It can only be by virtue of some *shared* difference through which they all interrelate.

³¹ I will argue that this restriction is unfortunate—all secondary qualities can be quantified. Kant restricts it to the strength of noumenal stimulation (and thus to “intensive magnitude”) only because he wants to tie it to the logical forms of affirmation/denial, which he applies *solely* to the notion of reality.

Recall that sense-consciousnesses are not aware of one another, that the scope of each sense-consciousness's awareness is limited solely to its own point-datum. Their separation is absolute, not yet grasped through a shared medium or **dimension of difference**. Because of this, sense-consciousnesses are not even aware of the *nature* of their separation from each other. Seeing the separation of sense-consciousnesses requires seeing how they interrelate through some transcending medium, and thus from a position that transcends and unifies them under one consciousness. The separation of *consciousnesses* from the (unified) me becomes the separation of *data* from each other.

This is a key point: “seeing” the plurality of sense-consciousnesses all together as one *is the same thing as* seeing the plurality of point-data all together in a unitary synopsis—which is the same thing as grasping their separation as a medium of interrelation. Sense-consciousnesses originate as separate, but their subsequent unification shows that their original separation was mediated by a single medium. We can call this unifying medium of sensibility **proto-space**. Kant calls it the **form of sensibility**. [B160 fn. 305] The form of sensibility is the mere difference underlying the plurality of sense-consciousnesses, not yet apprehended as a unity of sense data.

But this medium is not an object for consciousness until the imagination spontaneously combines these sense-consciousnesses by *drawing a line*. In order for a given sensation to become an intuition, it must be combined with all the others into the unity of what Kant calls *formal* space. The datum then becomes an object—i.e., an object for epistemic consciousness—and is called **intuition**. An intuition is consciousness of a datum in space. The two essential features of an intuition are *singularity* and *immediacy*.

Formal intuition: the emergence of the “I”

For Kant, every sense-consciousness is also a point-datum. Each point-datum has a formal aspect inherited from the form of sensibility, which we have called proto-space. The proto-spatial form of separation is innate, but the unified spatial field which we cognize is constructed. The **form of space** is simply the basis of separation that permits the reception of a plurality of point-data. I call this basis *proto-spatial* because the form of space does not present what we normally mean by *space*, which Kant calls **formal space**. Cognizing formal space depends on the activity of pure sensible synthesis, which combines the form of space (the proto-spatial elements) into formal space. “Space, presented as *object* (as we are actually required to present it in geometry), contains more than mere form of intuition; viz., it contains also *combination*, of the manifold given according to form of sensibility, into an *intuitive* presentation—so that the *form of intuition* gives us merely a manifold, but *formal intuition* gives us unity of presentation” [B160 fn. 305]. The mere form of intuition “contains as yet no *determinate* intuition at all. Determinate intuition is possible only through the consciousness of the manifold’s determination by the transcendental act of imagination (i.e., by the synthetic influence of understanding on inner sense)—the act that I have called figurative synthesis” [B154].

Kant identifies the act of figurative synthesis with the imaginary activity of *line-drawing*:

Thus the mere form of outer sensible intuition, i.e., space, is as yet no cognition at all; it provides only the manifold of a priori intuition for a possible cognition. Rather, in order to cognize something or other—e.g., a line—in space, I must draw it; and hence I must bring about synthetically a determinate combination of the given manifold, so that the unity of this act is at the same time the unity of consciousness (in the concept of a line), and so that an object (a determinate space) is thereby first cognized. [B137–38]

Kant here says that there can be a “unity of consciousness” in the cognition of space only if we traverse the plurality of sense-consciousnesses in imagination by drawing a line, and only if we draw the line by imagining the motion of a numerically identical point. This will now be explained.

Line-drawing: intuiting unity and formal space

“T”: the sensible unity of the knower

Sense-consciousness originates as a plurality. But epistemic consciousness is a unity. The first step towards cognition will be the combination of the plurality of sense-consciousnesses into the **intuiting unity of epistemic consciousness**. The plurality of sense-consciousnesses must become a plurality of received contents beheld synoptically by one intuiting consciousness. How can this happen?

The form of space: original medium of plurality

Kant posits the existence of a force internal to the knowing subject—a counter-force to the receptive capacity of outer sense. He calls it **spontaneity**. Spontaneity (or *act*) is Kant’s Leibnizian conception of the ontological subject as *conatus*.

The plurality of outer sense is combined by spontaneity. But I can combine data only if I have access to the gap-providing medium in which this combination is performed. The field within which I combine separated elements must be internal to me. An innate plurality can be combined into a plurality-in-unity only if the elements in the plurality share the same *basis or medium of separation*. Combination must occur as a *kind* of combination; the elements have to be separated in some *known* way. This way-of-separation can be known only if it is internal to the subject. So the subject brings to cognition not only activity of combination, but also the *form* of combination. Because it

is internal this form is also necessary, and a basis for necessary (intersubjective) truth. For Kant, epistemic apriority always rests on genetic apriority.

It may seem odd that a principle of separation can serve as a medium for unification that is distinct from our spontaneous acts of epistemic combination, which are directed by the understanding. But this is actually what Kant says: “For through this unity (inasmuch as understanding determines sensibility) space or time are first *given* as intuitions, and hence the unity of this a priori intuition belongs to space and time, and not to the concept of understanding (see § 24)” [B161 n. 305].

An outer sensible plurality can be a combinable plurality only if we reach-out to it with an inner (pure) sensible plurality that can accommodate it. Every kind of a priori combination presupposes an internal form of plurality, a dimension of difference *shared among all the elements of the plurality*. I can only combine a plurality that is internal to me. And when I combine elements internally, in the *res cogitans*, my knowledge of this combination is perfect.

Kant calls the shared difference that typifies the plurality received through outer sense the *form of sensibility*. It is the medium of separation in which the sensible plurality is originally received. Consequently, the form of outer sense is simply that kind-of-difference that all particular data (and their attendant sense consciousnesses) share. The form of difference that sustains the plurality of sense-consciousnesses in *humans* is the **form of space**. Since each sense-consciousness is also a point-datum, the combination of the former into a unitary “I” is simultaneously the combination of point-data into the simultaneous compresence of the unitary field of space. It is for this reason that a priori knowledge about space is possible.

Remember that Kant’s method will be to ground uncontroversial necessary truths in the process of knower-making, and then employ these grounds in the service of

justifying the controversial (for Hume) necessary truths referred to by the categories and necessary for the practice of mathematical physics. We have now seen that the truths of **geometry** are necessary because formal space is an outcome of knower-making—it is the correlate of the unity of the intuiting “I.” The imaginary process of knower-making that produces this synthesis is line-drawing. Kant’s theory of line-drawing will now be explained.

Line-drawing: synthesis of formal space is synthesis of the “I”

Intuiting consciousness is consciousness that is aware of the plurality of outer sense as a plurality-in-unity—i.e., as a unified spatial field whose extension derives from the shared difference that is the *form* of sensibility. This combination of sense-consciousnesses into the intuiting unity of consciousness means that the “I” has *itself* been unified according to the form of space. The form of space has epistemic necessity for Kant (i.e., geometry exists) because the form of space is just the nature of the shared, pluralizing separation by which a plurality of sense-consciousnesses come together into the “I.”

In order to bring the proto-spatial form of outer sense to the unity of the “I,” I must carry out the *synthesis of apprehension*. I must “run through” the plurality of point-data by imagining a *moving point*—that is, a point that I take as maintaining its identity through its continual movement through space and time. This moving point is the “figurative” analog of “the identity of act” [A108] that grounds in the ultimate unity of my internal unifying force of spontaneity, which always manifests itself as combinatory activity. The fundamental combining act is the act of spanning space with an identical point. The extension of the line is a record of the sensible plurality of outer sense *and* of

the passing of my inner states; the unity of the line is evidence of the unity of point/act that produced it:

Now, how it is possible that from a given state there should follow an opposite state of the same thing—not only can no reason make this comprehensible to itself without an example, but it cannot make this understandable to itself without intuition even. And this intuition is that of the motion of a point in space; solely the point's existence in different locations (as a succession of opposite determinations) is what first makes change intuitive. For in order thereafter to make even internal changes [in consciousness] thinkable, we must make time, as the form of inner sense, comprehensible figuratively through a line; and we must make internal change comprehensible through the drawing of this line (i.e., through motion), and hence we must make the successive existence of ourselves in different states comprehensible through outer intuition. [B292]

By moving “through” moments and spatial position, one and the same perduring point-entity can claim ownership of a series of points. But this point is posited (or drawn) by one and the same spontaneous agent, and so ownership by one and the same consciousness is verified as well. This is how a plurality of outer sense-consciousnesses becomes a plurality of places—i.e., by becoming a single, synoptic spatial cognition. The activity of line-drawing is the means by which the “I” verifies its ownership of every point in space; and it is the means by which the intuiting “I” first comes into existence. Prior to this, there was only a pure spontaneity existing alongside (as it were) the passing plurality of sense-consciousnesses.

Because every sense-consciousness is also a sense datum, bringing the plurality of sense-consciousnesses together into one actually accomplishes two unifications. On the side of consciousness (sensation, the inner), every sense-consciousness is combined with every other into the simple unity of the “I think.” On the side of the datum (intuition, the outer), every datum is combined with every other into the unitary but extended field of space.

The act of line-drawing is what appropriates (or refers) moments and points to a unitary intuiting consciousness. It expresses the subject's transcendence over space and time by verifying the subject's ownership of what will eventually be cognized as *position*.³² Prior to line-drawing, the points were ontologically different particulars; now, they are merely different positions of one and the same space (line). The unity of the spatial field lies in the fact that all of its constituent points all related to one and the same "I think." The necessity of intuiting unity is based on the following principle: that it must be possible to proclaim the truth of self-recognition at every point in cognized space. "The *I think* must be capable of accompanying all my presentations" [B131]. It is by tracing-over point positions that I touch them with activity—I *posit* the point; and through line-drawing, I gather them together under a unitary consciousness. It is only by doing this that the accompaniment of the "I think" at every point becomes possible.

This is the first step of synthesis—that of bringing the original plurality into relation with an identical intuiting subject. What were formerly sense-consciousnesses have become (objective) data that are related to one and the same subject of consciousness *and* related to each other by spatial position. With the generation of the "I think" through the unification of space through line-drawing, we have accomplished the first component of a fully functioning epistemic consciousness. We can call this **basic line-drawing**, for it must be distinguished from other acts of line-drawing—ones to which an *interpretation* has been added.

COGNITION (2): JUDGING UNITY

Sense-consciousnesses are plural in two ways. They are plural in their momentary presentation, and they are plural in their passing away. The former plurality, as we have

³² This will occur when space is divided and recombined by applying the form of logical quantification in ostensive judgment. See below.

seen, is unified into the synoptic unity of space. This is accomplished through what we have called *simple line-drawing*, which yields both the cognition of formal space as well as the intuiting unity of the “I think,” the first half of the structure of epistemic consciousness.

But through simple line-drawing we cognize no physical objects, just a momentary array of pixels. Physical objects are never present as real contents. They must be constructed, and for Kant they are constructed by imagining the pixel arrays as being connected in certain ways. These ways of binding are *forced* on the imagination by the **structure of ostensive judgment**—the “this (S) is P,” the second half of the structure of epistemic consciousness.

The structure of judgment analyzes into three structural components, which we have called the **components of judgment**. The components of judgment are the subject-position, the predicate-position, and the copula.³³ Their combination produces the unity of *atomic* judgment. Atomic judgment is the necessary **discursive condition** of epistemic consciousness, because consciousness cannot be brought into a unity that *knows*, that asserts truth, without subsuming an intuition under a universal. Thus the structure of ostensive judgment is identical with atomic judgment, which Kant calls the **categorical judgment**.

We will see that the components of judgment contain “rules” some of which are analogous to the operators of Aristotelian logic. These rules must serve three functions: (1) they must serve as the rules that guide the process of transcendental synthesis performed by the productive imagination, (2) they are contained in the components of

³³ By “subject” and “predicate” I mean only their *grammatical positions*, not the concepts actually contained in these positions. For the latter sense, I will use the locution “subject concept” and “predicate concept.” This is an important distinction because while general logic treats these positions by way of the universals they contain, Kant’s transcendental logic treats the way these positions themselves function as rules for combining point-data into physical objects.

judgment, which are the placeholders (grammatical positions) that “subsume” the products of this synthesis, and (3) they must serve as rules that the knower can use to emulate (or “schematize”) the process of synthesis, which is necessary to understand their sense.

Before we discuss these functions, let us exhibit Kant’s official tables of judgment-forms and categories:

Quantity	singular	particular	universal
Quality	affirmative	negative	infinite
Relation	categorical	hypothetical	disjunctive
Modality	problematic	assertoric	apodeictic

Table 1: Kant’s Table of Judgments [A70/B95]

Quantity	unity	plurality	totality
Quality	reality	negation	limitation
Relation	<i>inherence</i> –subsistence	<i>causality</i> –dependence	<i>community</i>
Modality	possibility–impossibility	<i>existence</i> –nonexistence	<i>necessity</i> –contingency

Table 2: Kant’s Table of Categories [A80/B106]

Function (1): providing rules of transcendental synthesis

The first function of the structure of judgment is to provide the rules that guide the productive imagination in its task of transcendental synthesis—its task of combining the continual passing of pixel arrays into a cognition of physical objects. Kant identifies these rules as the elements of the “this (S) is P,” which we have called the **components of**

judgment. The components of judgment are the subject, the predicate, and the copula.³⁴ The S and P both produce two types of synthesis, which Kant calls “mathematical” and “dynamical.” Mathematical syntheses are “directed to objects of **intuition** (both pure and empirical),” while the dynamical syntheses “are directed to the **existence** of these objects” [B110]. Mathematical syntheses produce awareness of aspects of the object that, being independent of time, can be presented directly in (spatial) intuition, while the dynamical syntheses produce some part of our awareness of how the “reality” of the object is determined in time itself.

The ways of synthesis and their corresponding components of judgment are as follows:

- **mathematical subject-position:** Our awareness of **magnitude** is produced by the forms of Quantity, which for Kant are forms belonging to the subject-position. These are the *singular*, *particular*, and *universal* judgment-forms.
- **dynamical subject-position:** Our awareness of **substance** as a substrate that undergoes changes of property is produced by the grammatical subject in “This (S) is P.”
- **mathematical predicate-position:** Our awareness of **quality** as a continuum of values is produced by the forms of Quality. These are the *affirmative*, *negative*, and *infinite* judgment-forms, which for Kant are forms belonging to the predicate-position.
- **dynamical predicate-position:** Our awareness of **property** as a content that varies (arises and passes away) is produced by the grammatical predicate.

³⁴ By “subject” and “predicate” I mean only their *grammatical positions*, not the concepts actually contained in these positions. For the latter sense, I will use the locution “subject concept” and “predicate concept.”

- **copula:** Our awareness of **causality** as the order of property-states in the objective time continuum is produced by the copula.

These will be spelled-out in detail in the next chapter.

Function (2): subsuming the products of synthesis under their grammatical positions in the structure of judgment

Asserting an ostensive judgment (one aimed at the passing plurality of outer sense) produces a kind of **grammatical intent**—that is, it engages an effort to find referents appropriate to the components of judgment. Since these referents are kinds of unity, they cannot be found in outer sense, even after it has been combined into the unitary field of formal space. As a result, the productive imagination is set in motion to compensate for this lack, being guided by the components of judgment in their role as rules of synthesis.

Each product of synthesis has a function of unity in judgment as its rule. Take the rotating cube example again. My effort to apply the subject-position to the (dynamical) passing-away of pixel arrays forces my imagination to imagine the points in the quadrilaterals as perduring through time: for each point on the cube, this point here-now is somehow *linked by identity* to some point from the previous array. As a result, instead of experiencing a sequence of separate pixel arrays, I “experience” a rotating, temporally identical body. This is the synthesis from which I abstract the category of *substance*. The other syntheses—those of body (magnitude), quality, property, and causality are produced in the same way: by the imagination being guided by the components of judgment in my effort to produce the referents these elements demand.

The next step is to subsume these unities under grammatical positions. It is no surprise that each kind of unity is subsumed under the very same grammatical element that served as its rule. Thus the substantial body of the cube that my imagination creates under the *guidance* of the dynamical subject-position is also that aspect of the object that I *subsume* under the same position when I assert an ostensive judgment, such as *This cube is rotating*.

This may seem trivial, but Kant treats it as a potential puzzle. He gives the example “All bodies are divisible” and points out that, under the rules of general logic, this can be rewritten as “Something divisible is a body” [A94/B129]. The problem is that both universals (*body* and *divisible*) can serve as either subject or predicate. Lacking any sufficient reason for mapping these universals onto the two grammatical positions, how can a determinate ostensive judgment be made? Kant’s misleading answer: we bring *body* under the category of *substance* and this forces us to map *body* onto the subject-position.

Actually, this problem of mapping universals only occurs in non-ostensive judgment. In non-ostensive judgment, the copula relates universals, and their placement in the subject- and predicate-positions can be reversed while preserving truth-value by using the appropriate logical modifiers (“all,” “some,” “is,” and “is not”). But in ostensive judgment, the copula relates substance and quality. If Kant had used ostensive judgments in his example, he would have seen that this subsumption under a category is unnecessary, because *body* is implicit in the subject-position of every ostensive judgment.

The grammatical positions of ostensive judgment are enriched by the syntheses that they guide *and then subsume*. Kant should have used ostensive judgments for his example. The ostensive versions would be: “*This* body is divisible” and “*This* divisible (thing) is a body.” We can see that what maps *body* to the subject-position is the subject-position itself. *Body* essentially belongs to the subject-position because the subject-

position automatically subsumes what it itself has made in its role as *rule of synthesis*. The referent of the subject-position is always a body. If I say, “This divisible thing is a body,” the referent of the subject is still, in fact, a body. In ostensive judgment, the universal I map to the subject-position is irrelevant. When I say, for any physical object that falls under *F*, “This *F* thing is *G*,” the *F* is merely a name. In fact, I can leave the subject-position semantically empty: “This is *G*” will do. Mapping universals only makes a difference in the predicate-position—as in “This leaf is green” vs. “This leaf is round.”

Interlude: generating the semantic categories through the process of reflection

The components of judgment contain rules that determine the imaginary products of synthesis. These products are then subsumed under the elements’ corresponding grammatical positions. Once I subsume multiple instances of an imaginary product-type, I am in a position to generate the semantic universal, or **category**, corresponding to it. This shows that the categories are not innate, but generated, or “acquired.”

For Kant, all universals are generated from intuitions through the **process of reflection**—comparison, reflection, and abstraction.³⁵ This holds for the categories just as much as it does for other concepts. To generate the category of substance, for example, I compare *this* referent of my subject-position, *that* referent of my subject-position, and so on ... and eventually abstract from many such referents what they all have in common—and this I call **substance**.

Similarly for the referents of my predicate terms. What do they all have in common? They all refer to **properties**—insubstantial, varying states that inhere in substances.³⁶ They also refer (in some cases) to what Kant calls qualities. A particular

³⁵ I will use the phrase “reflected under *F*” to indicate that some universal *F* has been generated by carrying out this process on some multiplicity of particulars.

³⁶ The terms *change* and *vary* are technical terms for Kant. Only substrates can change. When I say that *some thing* changes, I mean that there is one and the same thing that is *F* at one moment and *G* the next. I

property is (in some cases) taken as a value within a continuum of values. This value continuum is a second-order type, a **quality**. For example, a particular shade of green is really a particular value in a continuum of values. The property is the particular shade; the quality, the second-order concept *hue*. Every predicate can be quantified in this way; for example, *salty* contains an intensity—an object can be *more* or *less* salty. By comparing one second-order property to others (such as length, weight, elasticity, amplitude, velocity, duration, etc.), I eventually abstract the notion of quality as a value on a continuum of possible *other* values within the same second-order type and, for this reason, I can anticipate their change.³⁷

Finally, there is the question of how we abstract the category of **causality**. We compare referents of copulas, of statements of fact. The statement of fact brings a property into objective time, whose substrate is substance. This is just the relation expressed by the copula, “is.” We compare what all statements of fact have in common. They all bring properties into objective time. In objective time, properties arise and pass away in an objective time-order. An objective time-order is simply a *necessary* order—an order wherein state-value is *determined* by time-value. Objective time-order is irreversible. But this irreversibility *of time* only has empirical significance if it is expressed as a lawful sequence of properties. We notice that the order of events is necessitated by an empirical law, because only thus can we cognize the objectivity

measure this change *in reference to* some constant. Variation is the opposite—it is what is indicated by the *F* and *G*. What varies arises and passes away. Properties vary; substances change.

³⁷ Due to his desire to analogize the ostensive function of the components of judgment as closely as possible to the forms of general logic, Kant unfortunately reduces quality as second-order continuum *in general* to a second-order continuum of “intensity.” Thus a particular shade of red is, by Kant’s *a priori* mathematization, a value on a continuum, not of hues (wavelengths), but of opacity or brightness (amplitude). He does this because he ties intensity to reality, and reality to affirmation; and he does this in order to tie the notion of a quality-continuum to the Aristotelian logical operations of affirmation and denial, here treated as the poles of the continuum. We will deal with this and similar problems arising from Kant’s desire to correlate transcendental and general logic in Chapter 4.

(necessity) of the time-order. Under representationalism, an objective time order can only mean a *necessary* time order—that is, one that is necessitated by some rule. Kant gives the example of a house: I may apprehend its parts in any order, depending on the movement of my eyes. This subjective order is distinct from the *objective* order, which holds independently of my experiential route. A rule-necessitated time order is an irreversible one, and this irreversibility can only be expressed if empirical laws are in place where an *event* follows *event* necessarily.

In this way I generate the semantic categories of substance, property, quality, and causality.

COGNITION (3): INTELLIGIBILITY AND SCHEMATISM

The final function of the judgment-forms is to serve as rules of *schematism*. Schematism is Kant's word for synthesis that is carried out consciously. Synthesis is an occult process that is carried out spontaneously, or automatically. This means that it is carried out unconsciously. By Kant's criterion of apperception, however, it must be possible to carry out synthesis "with consciousness." This requires intentionally following the rule of synthesis as a procedure. We will treat the nature of schematism and its relation to automatic synthesis in Chapter 5.

Chapter 3: the Transcendental Deduction (A edition)

§ 13: WHAT IS A TRANSCENDENTAL DEDUCTION?

I have the right to use an *empirical* concept whenever an instance of it appears in sensation. But the categories *must always* apply to reality—their applicability is a priori by definition. But how can we *prove* this? How can this necessary and universal applicability be justified given that they have no empirical basis? Kant characterizes this problem as a question of *right*, or *quid iuris*.

- **Kant's question *quid iuris*:** How can we justify the employment of concepts whose objective reality is supposed to be a priori?

An argument showing such justification is called a **deduction**. Kant uses *Deduktion* in its Roman legal sense—as an argument intended to justify the legitimacy of a property claim by tracing the lineage of the claimant back to the original owner.³⁸ Kant's epistemological sense, a deduction is an argument that justifies the application of a universal to experience by showing that it traces back to an actual presentation. A **transcendental deduction** is a deduction that justifies the use of the categories by explaining how it is that they can refer to objects a priori. Showing this will be a challenge: the categories are concepts that *must always* apply in ostensive judgment, so their deduction cannot be one that traces back to empirical contents, which are contingent. What *must always* be true about experience cannot be contingent on empirical presentation.

³⁸ For a detailed history of Kant's notion of *Deduktion*, see Dieter Henrich, "Kant's Notion of a Deduction and the Methodological Background of the First Critique," *Kant's Transcendental Deductions*, ed. E. Förster (Stanford: Stanford University Press, 1989) 29-46.

What is the structure of a **transcendental argument**? Kant says that his analysis of space in the Transcendental Aesthetic was an example of such an argument: “We did earlier trace the concepts of space and time to their sources by means of a transcendental deduction, and we explained and determined their a priori validity” [A87/B119]. In that part of the Aesthetic, Kant defined “transcendental exposition” as

the explication of a concept as a principle that permits insight into the possibility of other synthetic a priori cognitions. Such explication requires (1) that cognitions of that sort do actually flow from the given concept, and (2) that these cognitions are possible only on the presupposition of a given way of explicating that concept. [B40]

In the Transcendental Aesthetic, this was carried out as follows: (1) If geometry is epistemically a priori, then space *must* be genetically a priori. (2) The necessary truth of geometrical propositions is self-evident. (3) Therefore, space must be genetically a priori. Finally, and most importantly, because space is genetically a priori, it is also a necessary condition of *appearing*: “only by means of such pure forms of sensibility can an object appear to us, i.e., can it be an object of empirical intuition” [A89/B121]. Nothing can appear unless it is already spatially situated. Since appearances are the content of reality, this means that the a priori science of geometry applies to real physical objects. This is Kant’s transcendental deduction of the genetic apriority of space. Following the definition above, Kant has shown (1) that that geometry can be a priori *only if* space is a priori, and (2) that space can be a priori only by being the innate form by which data (real or imagined) are received in outer sense.

The categories, on the other hand, are not conditions of objects being *given*; they are concepts that think the imaginary connections among point-data and thereby bring these syntheses to a unified cognition. It’s easy to see how the concepts of space and time have necessary application—the mere fact of *appearing* is sufficient to establish the

conformity of objects to Euclidean space. But mere appearing can occur independently of all other concepts, including the pure ones of understanding:

objects can indeed appear to us without having to refer necessarily to functions of understanding Thus we find here a difficulty that we did not encounter in the realm of sensibility: viz., how subjective conditions of thought could have objective validity, i.e., how they could yield conditions for the possibility of all cognition of objects. For appearances can indeed be given in intuition without functions of understanding. [A89–90/B122]

The goal of the First *Critique* is to show that the categories have a priori objective reference. This goal is reframed in the opening of the Transcendental Deduction as a question of right: *How can we justify the employment of concepts whose objective reality is supposed to be a priori?* The method will be to show that the categories are necessary conditions for having unified awareness of appearances. Only in this way can such a thing as “a priori objective reality” be established. But this method obviously leads to a new problem:

- **The question of the Transcendental Deduction:** How can concepts be necessary conditions of physical objects if concepts are not necessary conditions of appearing?

Showing how the categories can be established and necessary conditions of experience will be the primary task of the Transcendental Deduction.

§ 14: CONDITIONS OF PRESENTING AS LAWS OF PHYSICS

Physical unity is “thought” by a “concept”

There are only two ways to link a belief (subjective presentation) with an objective fact: “either if the object makes the presentation possible, or if the presentation

makes the object possible” [A92/B124]. The former method cannot establish a priori knowledge under representationalism, so Kant will defend the apriority of the categories by explicating them as necessary conditions for *presenting* physical objectivity. If there is such a thing as a necessary condition for presenting reality, then since reality *for us* depends on presenting, this condition will be prior to the object.

There are two conditions necessary for cognition of an object—the *intuition* that gives the object as appearance, and the *concept* that thinks the object’s unity. We have already seen how the “presentation of space” is a necessary condition for *intuiting* a physical object. *But how can a concept be a necessary condition for presenting the unity of a physical object?*

What does Kant mean by the “concept” or “form of thought” of a physical object? Recall the rotating cube example. Empirically speaking, all that is presented is a sequence of passing pixel arrays. But we “think” something else—a rotating cube. The difference, Kant says, is the effect of *understanding*. It is not intuiting but understanding that forces me to imagine the pixels as connected in just those ways that convey the formal unity of the rotating cube. This understood connection is what Kant means by the “thought” or “concept” of a product of sensible synthesis. These ways are determined by the forms of judgment, which in ostensive judgment act as rules of pixel-connection, i.e., as rules of synthesis. The concepts we call *body* (extensive magnitude), *substance*, *quality* (intensive magnitude), *property* (accident), and *causality* are rules of synthesis that “a priori precede [objects], as conditions under which alone something can be, if not intuited, yet thought as object as such” [A93/B125].

Kant's quick "explication" of the categories

At the end of this section, Kant abruptly "explicates" the categories: "they are concepts of an object as such whereby the object's intuition is regarded as *determined* in terms of one of the *logical functions* in judging" [A94/B128]. Categories are rules that make us imagine the object's intuition as determined by the logical forms of ostensive judgment.

He explains what he means with an infamously obscure example. Take the judgment, *All bodies are divisible*. Under general logic, "the understanding's merely logical use left undetermined to which of the two concepts we want to give the function of the subject, and to which the function of the predicate. For we can also say, Something divisible is a body." The assignment of *body* to the subject-position and of *divisible* to the predicate is arbitrary—I can easily reverse them by making the proper logical conversions: *Some divisible [things] are bodies*.

In ostensive judgment, however, grammatical positions are also *rules of synthesis* that force the imagination to combine point-moments in certain ways, thereby producing the "transcendental content" that is the formal unity of the generic physical object. These grammatical positions are not only rules of synthesis, they are also grammatical placeholders that subsume the transcendental content which they produce. The synthesis performed by a form of judgment *as rule* determines the ontological referent of the universal that has been placed in that position, and this forces that universal to belong to a certain *grammatical position*—i.e., the one that it already occupies.

For example, the subject-position directs (and subsumes) the products of synthesis that we refer to as *body* and *substance*. The subject-position is a rule that guides the construction of imaginary bodies and substances. In our example, it binds the pixels of a single time-slice into a contiguous *body* and also binds the succession of these body time-

slices into an imaginary *substance*. These products are automatically subsumed under the subject-position, which is the very rule that made them.

These products, body and substance, are constructed by the grammatical position *itself*, as rule of synthesis, and have nothing to do with the empirical concept that happens to fill this position. *Any* concept I place there will thereafter serve as a name for the object's body—even none at all. For instead of saying, “This leaf is green,” I can simply say, “This is green,” which still refers to the object's body.³⁹

“If, on the other hand, I bring the concept of a body under the category of substance, then through this category is determined the fact that the body's empirical intuition in experience must be considered always as subject only, never as mere predicate. And similarly in all the remaining categories” [A94/B129]. By this notoriously obscure remark, I interpret Kant to be alluding to the fact that we automatically “bring” *body* under *substance* because both *body* and *substance* are already contained as the intended compound referent of every ostensive subject term. The body of a physical object is the intended referent of the “this,” while the object as substance is the referent of “this S,” which refers to the transcendental object = *x*, i.e., the generic physical object as an empty collection of ways-of-unity.

We bring *body* under *substance* because we have to. Translators should have written “when” instead of “if” in the above quotation. The phrase “wenn ich den Begriff eines Körpers darunter bringe, wird es bestimmt” is better translated as, “When I then bring the concept of body thereunder, it becomes certain.” The irreversibility is the result of a necessity—a body is a transcendental content produced by the subject-position,

³⁹ We will see that the empirical concept is only important in the predicate-position because it there determines which of the various attributes of the object I am synthesizing as a value in a continuum. “This leaf is green” requires inventing a hue-continuum because I anticipate, a priori, the possibility of a change of color.

which also produces that of a substance. The concept “substance” is just the name we give to the “dynamical” referent of every grammatical subject, while that of “body” refers to the way a substance appears to me in a single moment—i.e., as mere spatial extension (the “mathematical” referent of the grammatical subject).⁴⁰ It is our recognition of this (a priori) subordination of *body* to *substance* that forces us to consider the “body’s empirical intuition ... as subject only.”

In other words, what makes the assignment of concepts to grammatical positions irreversible in ostensive judgment is the fact that, *in the physical world to which these positions refer*, properties are dependent on substances. This dependence relation cannot be reversed: properties are varying states *of* perduring substances and not vice versa. The referent of the subject-position in ostensive judgment is always the object as *substantial body*, and the referent of the predicate-position is always some state of this substance.

But if Kant had used a different example, such as “This leaf is green,” we could transpose it as, “This green thing is a leaf.” In this case, the subject concept is not already contained analytically in the set of concepts that have been abstracted from the what the subject-position points to in every generic physical object. In this case, the assignment really *is* a choice. We can now render Kant’s sentence in its more popular form as: “*Once* I bring the concept of a leaf under the category of substance, then through this category is determined the fact that the leaf’s empirical intuition in experience must be considered always as subject only.” The term *leaf* has been assigned the function of naming the object as body/substance by being placed in the subject-position. This is conditional necessity: I consider the referent of *leaf* as subject only because I have (by placing it in the subject-position) brought it under the category of substance. But this kind of

⁴⁰ I explain my interpretation of Kant’s mathematical/dynamical distinction in Chapter 2, “cognition (2): judging unity.”

conditional necessity cannot be what Kant means by “must be considered *always*.” In any case, it is uninformative to say, “If I place *F* in the subject-position, I must then consider the referent of *F* as the referent of the subject-position.” The dependence relation in ostensive judgment can only be the result of the way that the imaginary contents that we construct are related in experience. This, after all, is the definition of the synthetic a priori relation—two things being necessarily related in cognition of outer sense, where this relation is not one of logical subordination.

As we will see in our treatment of the Systematic Presentation, Kant’s a priori quantification of *time* and *quality* will ultimately render this ontological dependence as a mathematical dependence. Kant conceives the object’s state as one quantity (intensive magnitude) that is *determined by* another quantity (its time-position). We will then see that the irreversibility of subject and predicate in transcendental logic is due to the fact that the subject and predicate-positions (in ostensive judgment) function just like the independent and dependent *variables* in an algebraic function that determines state (the referent of the predicate-position) as a function of time (the referent of the subject-position). This is what finally determines the ontological relation of substance and property to which these positions (and so also the universals placed in them) refer.

The categories are the universals that we abstract from the kinds of synthesis, determined by the forms of ostensive judgment, that present the generic physical object. This is how a “concept” can serve as a necessary condition for presenting a physical object.

THE ESSENTIAL ARGUMENT OF THE TRANSCENDENTAL DEDUCTION

The Transcendental Deduction was entirely rewritten for the B edition. I will refer to the two versions as the A-Deduction and the B-Deduction. Both versions tell the same story, but with different emphases. The common story can be briefly outlined as follows:

1. Epistemic consciousness is unitary and its numerical identity must be verifiable; that is, it must be aware of its identity at every point-moment of experience. Consciousness that is self-aware in this way is called **apperceptive consciousness**.
2. When apperceptive consciousness relates to the passing plurality of outer sense with epistemic intent, it does so as *understanding*, through the act of ostensive judgment. Understanding can access sensibility only by means of ostensive judgment.
3. Ostensive judgment cannot apply to sensibility unless there exist referents for its essential syntactic elements, the *forms of judgment*.
4. These referents are not given in intuition. They are types of spatiotemporal combination carried out by the imagination. The imagination carries out acts of *transcendental synthesis*, governed by the forms of judgment in their function as *rules* of this synthesis.
5. So epistemic consciousness must imagine point-moments as being connected in certain ways, i.e., in the ways demanded by the forms of ostensive judgment. *These connections are thus necessary conditions of epistemic consciousness and experience.*
6. These ways of connection constitute the necessary imaginary forms of the generic physical object. The categories are just the semantic universals that we generate from these forms through the process of reflection.

OPENING OF THE A-DEDUCTION (A95–98)

The necessary features of any cognition-friendly world

Kant begins the A-Deduction by outlining the most basic features that an a priori concept must have. First, an a priori concept must be open to empirical content if it is to hold for reality since reality *for us* is comprised of empirical point-data. Second, in order to do this, these concepts must be empirically empty. Third, these concepts must apply everywhere and always to these empirical data.

Before we can examine candidate pure concepts, we need to ask what it is that could serve as referents of concepts that apply everywhere, always, to all data, while being themselves non-data. Kant's clever answer is that the only concepts that would definitely apply to all of sensible experience and knowledge are concepts that are necessary conditions of such knowledge. **But how can a concept be a necessary condition of experience?**

Leaving aside the mystery of how a concept can be a condition of objective cognition, Kant isolates the question of necessary conditions and attempts to answer it by means of a thought experiment. If we close our eyes and try to imagine wildly different possible worlds, what are the ways-of-being that we must retain in order to say that we *experience things that are facts* in these worlds? In other words, what are the necessary conditions (if any) of a cognition-friendly world? The experiment is carried out by abstracting from everything accidental and particular in a physical cognition and then itemizing what remains. What remains is a *pure physical world*: space, time, and entities that exhibit the kinds of unity required for them to be *facts* that conform to the subject–predicate relation. We can imagine this world as a space-time containing some number of transparent, indefinitely shaped substances interacting under causal necessity whose properties (if they had any) would be continuously changing values in a second-order

continuum. These, then, are the “pure a priori conditions of a possible experience and of an object of possible experience” [A96].

Concepts as conditions of physical cognition

Kant then announces that these conditions of cognition-friendliness *are concepts*, specifically, they are the categories. “And if we can prove that only by means of the categories can an object be thought, this will already suffice as a deduction of them and as a justification of their objective validity” [A96–97].⁴¹ Under representationalism, this can only be done from the side of the subject—i.e., from the side of understanding and “something more” that brings real point-data into conformity with understanding.

Reality consists of point-data that are *given*. These, the contents of reality, cannot be replaced by innate concepts. How, then, can innate concepts find their way into reality if there is no “room” in the contents? Well, we have just seen that the pure physical world subsists independently of empirical contents—it is a world of pure relations. The essence of physical cognition is relational, relations are not empirical contents, and this fact provides an opening for realizing our innate concepts. We realize our innate concepts by inserting them *between* these contents—as their (imagined) relations.

This is how Kant solves the Herz problem of how spontaneous products of the understanding can have objective reference since real objects are given externally to the knower. Innate concepts can have objective reality because what they refer to are not empirical contents, but kinds of relation. And innate concepts *must* have objective reality a priori because these kinds of relation are necessary conditions of cognition-friendliness.

⁴¹ Kant does not remind the reader that by “pure concepts” he means “forms of judgment.” This identification has already been made in the Metaphysical Deduction. In fact, throughout the Transcendental Deduction, Kant will often use “category” instead of “form of judgment.” The categories are not original rules of synthesis, but universals that are acquired by comparing the products of transcendental synthesis ruled by the forms of judgment.

The form of the pure physical world is the way it is because it is demanded by unitary epistemic consciousness. Pure consciousness is simple, but not when it intends to *understand*. This forces it to reach out to sensible plurality through the structure of judgment, which is itself plural and combinatory. Data are combined into the subject-position, data are combined into the predicate-position, and these combinations are themselves combined into the final unity copula, thereby giving the data the unity permitting them to interface with a unitary knower. Kant conceives the structure of judgment as a two-pronged combining mechanism that brings sensible plurality into the unity of epistemic consciousness, with the subject and predicate-positions acting as the prongs, and the copula as their combination. The subject-position rules (and subsumes) the production of body and substance, the predicate-position does likewise for the production of quality and property, and the copula then brings these together into causal determination, by determining property as a particular quality-value according to substance as continuum of time-values.

These forms of ostensive judgment, which combine (and think) the unity of the pure physical world, Kant announces, “we find to be the categories. And if we can prove that only by means of the categories can an object be thought, this will already suffice as a deduction of them and as a justification of their objective validity” [A96–7].

Part of this proof was provided in the Metaphysical Deduction, where Kant argued that judgment is the combining mechanism that brings plurality into unitary epistemic consciousness. It is the effort to understand by means of judgment that brings “the *pure synthesis* of presentations *to* concepts” [A78/B104]. Synthesis alone does not yield cognition until it has been brought *to concepts* by means of the understanding. To do this, we need the pure concepts that “give *unity* to this pure synthesis and which consist solely in the presentation of this necessary synthetic unity” [A79/B104]. Kant

conceives these pure concepts as functions that unify presentations into the unity of judgment—i.e., into the unity of an assertion or truth-claim. And the same functions of unity that realize my effort to know through the combination of the components of judgment into a *judgment* also force me to imagine the referents of these elements as being spatiotemporally combined in certain ways. Otherwise, my judgment would have no objective reality. This proves that we can “think” objects only by means of the categories, which are the semantic universals that refer to (and are abstracted from) the products of the necessary syntheses ruled by forms of judgment.

What is missing from this description is what is taken-up in the Transcendental Deduction. The first is a *general theory* of the process of synthesis that imparts the unity of the form of judgment to the imagination. How exactly does this occur? This question is answered by Kant’s theory of threefold synthesis—the syntheses of apprehension, reproduction, and recognition. In other words, we still do not know how grammatical positions can serve as rules of synthesis.

The second thing needing explanation is why it is that the world hangs together as it does (in a way that always already accords with the unity of possible judgment) even when I’m not making ostensive judgments? This question is answered by Kant’s principle of absolute necessity, the necessarily verifiable numerical identity of consciousness *across all experience*. While this is not explained clearly in the A-Deduction, it is clear from the examples in the B-Deduction that this act can only be made intelligible as an act of line-drawing, performed while “attending” to a certain feature of the act. In any case, the purpose of the Transcendental Deduction is not to explain the rules, but to explain how such a thing as necessary objective unity is possible.

THE THREEFOLD SYNTHESIS: SYTEMATIC EMPIRICAL VERSION (A115–16)

Kant expounds his threefold synthesis three times in the A-Deduction. The first one is a lengthy exposition in Section II, from A98–106. The others are in Section III—a top-down exposition at A116–119, and a bottom-up exposition at A120–24. Kant says that Section II is only meant to “prepare the reader” for the systematic expositions in Section III. I will follow the systematic top-down exposition and make reference to the details from Section II when necessary.

Kant begins his “systematic” account by listing the steps we would expect to find in a British-empirical account of concept generation: (1) empirical sense permits **perception** (consciousness of the succession appearances as such), (2) empirical imagination permits **association** (links between data that force the imagination to produce one datum when another is given or posited), and (3) empirical apperception presents some data as having **generic identity** (as falling under one and the same universal).

Kant will argue that these three steps actually depend on acts of knower-making, i.e., on acts of pure spatiotemporal synthesis that result in unitary epistemic consciousness:

Sense perception

I am only aware of one point-moment at a time, so I can perceive a multiplicity of data only by apprehending these data serially. Perceiving *a* multiplicity means being aware of the medium that contains them—the unity of a collection is the containing field in which it subsists. So Kant says that a “manifold would not be presented as such if the mind did not in the sequence of impressions following one another distinguish time” [A99]. Before I can cognize a collection as such, I must be aware of the medium that

transcends and contains this collection. The only synthetic medium for consciousness is space.

Imaginary association

To say that two concepts associate is to say that when *a* is presented, then the imagination will be forced to produce an image of *b*. The concepts *a* and *b* become associated by being frequently or always presented together in experience. This can happen only if physical objects are constituted so as to present only certain combinations of data, that is, only if datum *a* is consistently presented with datum *b* (and only certain others). Sadly, Kant does not give a pure synthesis that can account for this kind of empirical reproduction. The best that he can do is to *point out* that consistent combinations of only certain data depend on physical lawfulness, and then *add* that whole physical objects cannot be cognized unless the point-data of apprehension are linked together in certain ways—so that one follows from the other, but in a way totally different from the following-by-association he was trying to explain. The “following” here is the necessary following that occurs *in the act of apprehension itself*. Data must be apprehended so that they are linked into (1) spatially contiguous body-points and (2) temporally continuous substance-moments.

Conceptualizing generic identity

Empirical apperception (the ability to recognize that appearances at different places in space and time share an identical property) would be impossible if all appearances did not belong to the same consciousness. As we will see, for Kant the condition that experiences all belong to the same consciousness is not trivial because it must be *verifiable*—it must be possible for me to be conscious of the identity of

consciousness across all the dimensions of difference of which I am aware *and verify this*.

THE THREEFOLD SYNTHESIS: TOP-DOWN VERSION (A116–19)

Necessary objective connections as necessary conditions of apperceptive consciousness

Kant illuminates the “inner basis” of these syntheses by starting from pure apperception—he will move from apperception downwards to the passing plurality of outer sense. To understand the way things *must* be combined, we need to understand them under the “highest” unity into which all combination must *ultimately* conform—the *teleological* unity that flows from the absolute necessity that all data must belong to one and the same subject: “We are conscious a priori of the thoroughgoing identity of ourselves in regard to all presentations that can ever belong to our cognition, and are conscious of it as a necessary condition for the possibility of all presentations” [A116].

All my cognitions belong to *one* unified (biographical) experience. This entails that the “I” (the knowing subject) must be numerically identical in all my cognitions. And this, finally, entails that the knowing subject “see” them as being connected in certain necessary ways. Kant calls this “the *transcendental principle of the unity* of whatever is manifold in our presentations.” We can call this the **principle of apperceptive unity**.

The principle of apperceptive unity is the discovery that will explain the possibility of necessary unity in physics. In order to establish necessary connections within representationalism, Kant needed to find some indefeasible basis of necessary unity. The necessary unity of verifiably identical consciousness (which we may call **apperceptive consciousness**) is this basis. But necessary unity of consciousness can serve as the basis of necessary unity in physical objects only if the former can *produce*

the latter. Well, under Humean data-sensualism, which Kant accepts, the unity of object *can only be produced by the subject*—i.e., through imaginary synthesis. The key difference is that, for Hume, the natural relations that direct the imagination do so *independently* of unitary consciousness, which is simply an unexplained given. For Kant, this unity is achieved and must be explained. Moreover, it is an active unity—one that functions as a *power* that can stamp its nature on cognition by connecting point-moments in acts of synthesis: “the possibility of the logical form of all cognition depends necessarily on the relation to this apperception *as a power*” [A 117]. The Humean subject is a ready-made unity, and as such can make no demands on the imagination, whose ruling forces flow from empirical regularities. The Kantian subject arises as a plurality of sense-consciousnesses, which must be unified—and unified according to the forms of judgment in case its intent is epistemic. **But how does the unity of the knower “flow” into the unity of the object, whose matter is sensible?**⁴²

Apperceptive unity flows into the object by means of synthesis. Kant conceives this flow as follows. Relating elements to a single item also relates these elements to each other. Consequently, relating elements to a single item *necessarily* also relates these elements to each other *necessarily*: “For any such presentations present something in me only inasmuch as together with all others they belong to one consciousness; and hence they must at least be capable of being connected in it” [A116]. It is necessary that all data be *brought* to an identical consciousness, and this act of bringing also *connects* these data with each other. Thus the demand that all data have the same knowing subject entails that

⁴² This is how the Transcendental Deduction reformulates Kant’s famous question to Herz into more manageable terms. Kant’s question to Herz was, How can concepts that have their origin in our minds apply to objects that are given? Here, the question has been restated in terms of *unity*: How can the *indefeasible* unity of the knowing subject find its way into the *problematic* (for Hume) unity of the generic physical object?

these data also be connected to each other in certain ways. These certain ways are the necessary connections to which the categories refer.

Kant is not yet telling us what these connections are. His modest aim in the Transcendental Deduction is only to offer a way for explaining how necessary connection is possible—i.e., by establishing the thesis that bringing data *to a unitary subject* also connects these data *with each other*. Kant's modest argument:

1. For data to belong “to” one consciousness, they must all be present together “for” that consciousness, and so they must all be related to each other “in” that consciousness. In other words, combining sense-consciousnesses *into* apperceptive consciousness places sense data *under* a synoptic gaze that beholds their relations. Some of these connections are contingent; some are necessary. The necessary ones are the referents that we assert as necessary truths—i.e., as synthetic a priori judgments. Kant's position is that necessary objective connections are necessary because they are necessary for bringing data to apperceptive consciousness. *Bringing data to understanding entails understanding them as being connected in certain necessary ways.*
2. Combination is never given, but is carried out internally by the subject's power of productive imagination. This is possible because the matter of the objective reality is nothing but outer appearances. Since combination of sense data into physical unity is carried out by the subject, *it is possible for the necessary unity of the subject to “flow” into that of the object.*
3. Combination into apperceptive consciousness means combination into a unitary *thought*. Consciousness of a linked (reproduction) collection (apprehension) is still not consciousness *of a unity* unless it sees what all the linked elements in the collection *have in common*. Seeing this, Kant says, can only happen by seeing the “rule” that administers the collecting and linking as so many steps in a unitary procedure. Thinking the unity of a plurality of data requires collecting and linking them under a unitary *rule*.
4. There is a highest rule over all others, one that is indefeasibly necessary, and which is the basis of all other necessity. This is the principle of apperceptive

- unity. Necessary connections in the object must ultimately trace back to (or: have their *Deduktion* in) this supreme unity.
- CONCLUSION 1: “Now the unity of the manifold in a subject is synthetic; therefore, pure apperception provides us with a principle of the synthetic unity of the manifold in all possible intuition”—a principle for at least legitimating the necessary connections that we are familiar with. [A117]
 - CONCLUSION 2: The presentation that makes the object possible is the connection of the spatiotemporal positions of appearances by the imagination. These connections are necessary features of objects because they are necessary conditions of apperceptive consciousness—of the knowing subject’s ability to verify its identity at every point-moment. “Therefore the principle of the necessary unity of the imagination’s pure (productive) synthesis prior to apperception is the basis for the possibility of all cognition” [A118].

It is apperception *as epistemic* that guides productive imagination

The next question Kant must answer is: What are the *rules* of this synthesis? This is addressed at A119, a turning point in Section III: “*The unity of apperception [considered] in reference to the synthesis of imagination is the understanding; and the same unity as referred to the transcendental synthesis of imagination is pure understanding.*”

Imagination has no internal control of its own; it makes images under the coercion of some power that is external to it. When apperceptive consciousness reaches out to the plurality of outer sense, it does so *as understanding*. Understanding only occurs in the shape of *judgment*. In realizing this, Kant has discovered the rules of transcendental synthesis. These rules are the logical forms of judgment. The products of synthesis are the kinds of spatiotemporal connection denoting the pure physical object, i.e., the kinds of spatiotemporal connection we reflect as the categories. Thus Kant’s final conclusion: “Thus it follows that pure understanding, by means of the categories, is a formal and

synthetic principle of all experiences, and that appearances have a *necessary reference to the understanding*” [A119].

THE THREEFOLD SYNTHESIS: DETAILED BOTTOM-UP VERSION (A119–24)

At the end of A119, Kant explains the stages of the threefold synthesis from the bottom-up in order to show the metamorphosis from what is originally given (a plurality of sense-consciousnesses) to physical objects whose point-data are connected according to the ways reflected under the categories.

Apprehension

Appearance cannot be “combined with consciousness” in the facile way favored by empiricism for the simple fact that “every appearance contains a manifold, so that different perceptions are in themselves encountered in the mind sporadically and individually,” thus they “need to be given a combination that in sense itself they cannot have” [A120]. Appearances are many, epistemic consciousness is one. So the imagination performs the operation of apprehension on this manifold, the goal of which “is to bring the manifold of intuition to an *image*” [A120]. Point-data are given *as* point-data, but must be melded together into a spatial figure, or image.

Reproduction

An image, properly speaking, is not a collection of points, but a unified whole. Yet a collection of points is just what *mere* apprehension would yield. What is missing is “coherence.” The points in a proper image are not just contiguous, they are also *linked* to each other in a way that presents a whole.

Recall that for Kant the archetypal act of synthesis is *line-drawing*. When I apprehend two points in the act of line-drawing, I do so serially—first point *a*, then point *b*. But for these points to cohere so as to present a *whole line*, I must be conscious of

something more than consciousness of *a* “and then” consciousness of *b*. Rather, the movement from *a* to *b* must be “necessitated”—so that when I posit *a* I am *propelled* to posit *b*. Not only that, when I arrive at *b*, I am forced to imagine *a* as having been “brought over” into unity with *b*. This is the force of the reproductive link, which allows me to carry out the “bringing over” that is necessary for presenting an organic unity. Reproduction is what gives apprehension its internal directedness of propulsion. The melding together of point-data by apprehension cannot occur without a force that “holds” these points together. This holding-force is nothing other than my compulsion, when drawing a line, to *apprehend point-data in one direction rather than another*. From a point I can move in any direction. The rule is what determines my movement, and hence what makes “the reproduction of the manifold necessary a priori” [A105].

Recognition

But the only thing that could propel my awareness from *a* to *b* (and back again) is a **unitary procedural rule**, such as “I am drawing a line by imagining a moving point.” Thus the unity of reproduction, which only propels the mind *between* point-data, is itself propelled by a force that transcends *all* the point-data and unifies them as posits of *one* act. Every imaginary line I draw “extends” my numerically identical transcendental self as *act* (as the spontaneity that realizes, literally, my transcendental self) across space and time, and thereby classes all the points traversed in the extension with one apperceptive consciousness.

Kant’s argument is that whatever I take-up into my imagination through the act of apprehension can only become “classed with one consciousness (original apperception)” if its inter-point links are posited by one and the same rule. Ordinary rules of line-drawing have only hypothetical necessity—they link the points together into the form of

some particular figure. But *every* datum in my experience is connected to every other in some way—this red patch is *darker* than that red patch, or this red patch is larger than (or 5 inches to the left of, or 10 minutes after) that patch. These world-wide relations, being all-encompassing or universal, can, under representationalism, only be accounted for by the one thing that all data *must* have in common by indefeasible necessity. “This basis ... we cannot find anywhere except in the principle of the unity of apperception in regard to all cognitions that are to belong to me” [A122]. The numerically identical subject is the basis of all universality. In order for data to be brought to this identical subject, they must be connected by the rules of unity contained in the structure of ostensive judgment: “all appearances must without exception enter the mind or be apprehended in such a way that they accord with the unity of apperception.”

RULE AND SYNTHESIS

The primary mystery surrounding Kant’s construction of physical nature is the link between *understanding* and *sensibility*. This should be no surprise—it is the way he has imported into the First *Critique* the problem of the a priori reference of *concepts* to *objects* that motivated his letter to Herz. In his letter, Kant is troubled by how it is that innate concepts can apply to given objects a priori. His solution is to define the pure concepts as innate ways of relating the subject- and predicate-positions in judgment. In non-ostensive judgment, this is the relation of logical subordination. But in ostensive judgment, this is the relation between empty placeholders that become filled by the ways-of-combination contained in the generic physical object. Sensibility alone contains no referents for the subject- and predicate-positions, so these very positions act as rules of combination that force the productive imagination to *make* these referents.

In the Transcendental Deduction, the interface between concept and object becomes rendered as the interface between *apperceptive consciousness* and *time*. The unity of the former is indefeasible—all of my experiences must be (verifiably) mine. The unity of the latter, thanks to Hume, is problematic—do point-data *really* have to be connected in these physical ways? Kant conceives both the unity of the subject and the unity of time as containers—as kinds of plurality-in-unity: “all consciousness belongs to an all-encompassing pure apperception just as all sensible intuition belongs, as presentation, to a pure inner intuition, viz., to time” [A123–24].

Recall the “presupposition” that Kant tells us to keep in mind at the opening of the detailed version of the threefold synthesis—that being aware of data in time entails being aware of how they are all “ordered, connected, and brought into relations” in time. [A98] In order to be a container of sensible plurality, the data *in* this container must be connected in the ways necessary to mesh with an epistemic (experiencing, recognizing) apperceptive consciousness. (It must also be able to accommodate whatever activities are required for the latter to verify its numerical identity across all points and through all moments.⁴³)

The only true unity is the unity of “all encompassing pure apperception.” In the face of passing plurality, this unity must be verified as one that is numerically identical everywhere and every-when. But this necessary unity of apperceptive consciousness can descend into the sensible manifold *only as a rule*—a rule of apprehension-reproduction, which in the A-Deduction means a rule of time-binding.⁴⁴

⁴³ These latter are acts of “figurative synthesis”—acts (as we will see in the B-Deduction) that Kant identifies with *line-drawing*.

⁴⁴ In the A-Deduction, time is the primary form of intuition. Kant’s reasoning is that inner sense is “larger” than outer sense, since inner sense includes not only what is presented in outer sense (that takes only a moment), but also non-spatial sensations, such as somatic data. In the B-Deduction, however, this priority is reversed. This is not surprising, since the B Edition was written mostly as a refutation of claims that Kant’s system was a full idealism as opposed to a merely formal idealism, i.e., one whose ideal elements

So the real interface between understanding and sensibility plurality is found in the interface between **rule** and **synthesis**. The rule is a unity that subsists in the fiat of apperception; synthesis, “although performed a priori, is yet always in itself sensible, because it combines the manifold” [A124]. This is why the syntheses of apprehension and reproduction by themselves never actually result in a unity.

RULE AS “CONCEPT”

The highest and final unity, *towards which the preceding sensibility-based syntheses have been geared*, is the unity of recognition “in a concept.” This means two things.

First, remember that Kant has framed his expositions as answers to problems of missing unity in the empiricist account of concept acquisition. So the concept-unity being explained is the ability to generate universals through the process of reflection. This is the **unity of the universal**—the unity of universal generated through the process of reflection, which requires the ability to *compare* objects in *one consciousness*.

What the empiricists failed to explain was how the capacity for comparison necessary for the process of reflection is possible. It is possible, Kant says, only if multiple physical objects can be beheld in one consciousness. This is Kantian **transcendental unity**—the unity of the physical object, which is both the “object = *x*” and the all-encompassing unity-of-experience that allows one knowing subject to move back and forth between individual objects in the imagination, which is necessary for comparison.

were restricted to forms of judgment, space, and time. Kant refutation of idealism is carried out by arguing that, *when it comes to our capacity for time-determination*, space is prior to time. Both Editions can thus be harmonized as follows. In terms of *maximal containment*, inner sense is prior to outer sense—unifying things spatially does not unify them temporally. But in terms of *intelligibility and presentation*, space is prior to time. Time is intelligible only as a line, and acquires its sense only through the act of *drawing* a line, and this depends on space.

Kant unfortunately calls both kinds of unity “recognition in the concept.” This has led to countless interpretive difficulties. The solution, as we noted in Kant’s other attempts at demonstrating an empirical/transcendental analogy, is to see the two kinds of unity, not as analogous or isomorphic, but as in a dependence relation which overstates the analogy by using the same terms. There are both empirical and transcendental syntheses of apprehension, of reproduction, and of recognition. But the transcendental “counterparts” of the empirical syntheses are all varieties of mathematical construction, and bear no resemblance to the empirical stages of concept acquisition having the same names.

Chapter 4: the Transcendental Deduction (B edition)

§15—ON THE POSSIBILITY OF A COMBINATION AS SUCH

B129: combination is mandatory for knowledge

Combination is mandatory for knowledge. For example, **truth** is the intended *correspondence* between belief and objective fact. I can justify this correspondence (and thereby acquire knowledge) only by comparing my fact-representing product (judgment) to the objective fact that it intends to emulate. I can compare two things only by combining them in thought. A **judgment** is also a kind of combination—to assert that “S is P” is to combine *S* and *P* in the copula. Finally, the object of knowledge must also be a kind of combination. The object of knowledge is either a **logical object**, which is a logical combination of universals (possible predicates), or a **sensible object**, which is a physical combination of point-moments. The object of knowledge must be a kind of combination because an object of knowledge is an object of *judgment*, and its essential combination.

A problem arises when we consider the relation of knowledge to *reality*. What is reality? For Kant, reality is what overwhelms the subject with a continually passing plurality of immediate contents that have the “form” of point-moments since they are received by the pluralizing forms of space and time.⁴⁵ Since these contents are immediate, sensible consciousness *itself* arises as a plurality. Consciousness of reality is originally plural, yet knowledge involves unity necessarily, and I *am* a knower. This is what I have called **Kant’s problem**.

In order for me to be a unitary knower, this unity must *arise*, and it must arise *for the subject*. And so Kant will argue that the unity necessary for consciousness-of-

⁴⁵ This is pre-epistemic consciousness, what Kant calls “presentation without consciousness” (a possibility borrowed from Leibniz that we will discuss in our treatment of § 16, below).

plurality can arise from the given sensible plurality *only when the subject itself combines it*. All combination, even the unity of the object, is the effect of the subject's effort to understand. Thus Kant attributes the presentation of combinations to the activity of the **understanding**.

B130: all combination is an act of understanding

As already discussed, for Kant there are only two subjective powers directly involved in experience—passive sensibility and active understanding. The passive and sensible subject presents an *uncombined* passing plurality of point-moments. This is its only power. Consequently, combinations can only be presented by the subject's *other* power of presentation—i.e., by its internal active (spontaneous) powers of understanding and imagination. Combination is “an act of **spontaneity** by the power of presentation” [B129].

By combination Kant means any way of separation-and-connection that I understand. For example, when I see two spots in space, I am aware both of their being separated (by some magnitude of spatial *distance*) and also of their being together (as a pair of spots *in* the same space). According to Kant, any understood combination is one that has been made by the understanding itself: “all combination is an act of understanding—whether or not we become conscious of such combination; whether it is a combination of the manifold of intuition or of the manifold of various concepts” [B130]. A meaningful *content* can originate as something given, but a *unity* must be performed.⁴⁶

⁴⁶ Note that while consciousness of combination is optional, its being produced by self-activity is not. My awareness of my combining activity may be full or dim, but my carrying it out is mandatory, for understanding can only analyze complexes that it itself has made, since sensibility presents *no* combination.

All combinatory acts of synthesis are carried out as a consequence of the subject's effort to understand. An object for Kant is any unified complex that can serve in the subject-position of a judgment. An object can be a logical combination of predicates, or it can be a physical object, which is composed of various kinds of imaginary ways of combining point-data so as to bring the object into the kind of unity that we need in order to reconstitute it in imagination as a logical complex.

Kant says that synthesis is carried out whether or not I am conscious of it. How can we know that unconscious synthesis occurs? Take the visual presentation of a green chair. This can happen without consciousness; for example, I could stare at the chair while daydreaming about something else. In that case my sensibility would present an appearance of which I am yet unconscious. But I can *become* conscious of this object. And when I do, Kant notes, I find that I am presented with an object that *automatically falls apart* into just the elements I need in order to reconstitute it myself as a judgment that I can assert. I can say, for example, "This chair is green." Kant's point is that an object could be so accommodating to my innate *conscious* power of re-presenting it in a judgment only if it had already been originally presented by the same power. That is, I (as conscious subject) could separate and recombine the object the way I do *in a judgment* only if I (as unconscious subject) have already separated it and combined it in its original presentation.

When I become conscious of some particular green chair, I notice that it automatically breaks apart as a logical combination of elements that I can pay attention to individually, such as *green, chair, plastic, glassy-smooth*, etc. These are potential predicates. I am also aware of the object itself as a whole, as the collection of all these

characteristics.⁴⁷ This collection of all the object's marks is the referent of the subject-position. The object thus comes ready-to-be-parsed into the structure of judgment, "This (S) is P." All I have to do is consciously separate-out one of the characteristics and then consciously reattach it. Doing this is identical with the act of judgment.

Thus all objects are combinations by understanding, even when I am not aware of them; and I know this because I can be aware of them and, when I am, I find them to be understandable—i.e., I find that the combinations "already in" the object are just the ones that I can emulate in imagination. I can make my own imaginary object by affecting my own intuition myself and making an image that is also a logical combination. Being "aware of" entails being "aware that"—i.e., it entails that I call on my awareness itself as a source of information and make an imitation of the object of my awareness. I am aware (consciously) that "S is P" only if I am aware of the object as a presentation of an S that is P. And being aware of the objects as a logical combination, because it is combination, could only be possible if I was the combiner.

Kant's claim is that the object could automatically fall apart along the logical fault lines required in order to make analytic judgments about it only if it had been put together *in order to fall apart that way* by the understanding, which is our capacity to judge. This shows that that act of combination by understanding preconditions all possible data of awareness; otherwise objects would not be a priori amenable to the structure of judgment.

Thus the understanding that consciously asserts by means of judgment is the same understanding that presents the object as a logical complex of *green* and *chair*. This permits recognition and imitation, and thereby permits truth, which is just the correspondence that an imitation enjoys with what it imitates. When I verify this

⁴⁷ The object as a whole might itself be an instance of a universal—a physical object kind. The object kind is also a possible predicate, but since it is meant to refer to the object, it belongs in the subject-position.

correspondence, I have knowledge. Truth-in-understanding (or knowledge) is possible only if both judgment *and* object are combinations carried out by the subject's understanding. For Kant, objects are amenable to being understood because *they are themselves constructed by understanding according to the form of understanding*. This form is the structure of judgment: "S is P."

B130: combination not given, must be made by (prior) self-activity

The only way that I could emulate a plurality of sense-consciousnesses in a true judgment is if the object itself is understandable. But, as with any combination, objective combination can be understood (and thereby emulated as judgment) only if it is carried out by me. The only way I could emulate objective combinations by means of judgment is if I myself have *previously* combined these elements into the object *according to the same structure of judgment* that emulates it. I can only understand combinations that I can myself perform because understanding is combining: "we cannot present anything as combined in the object without ourselves' having combined it beforehand" [B130].

Kant locates the source of unity in the activity of the subject because sensibility delivers a plurality. Locating the source of unity in the pre-synthesized (noumenal) object would accomplish nothing, for the object can only become presentation *for me* by stimulating my sensibility, whereupon it becomes atomized into point-moments. Hence: "among all presentations, *combination* is the only one that cannot be given through objects, but—being an act of the subject's self-activity—can be performed only by the subject himself" [B130]. The object can only become presentation *for my* unity by being synthesized by *me*.

In fact, it is in *virtue* of the fact that sensibility *lacks* combination that a priori knowledge is possible. I understand *by means of* combination. Unless I combine, I cannot

understand. Locating combination in the pre-sensed object would do no good for a candidate knower, who understands only by combining.⁴⁸ In fact, if the object *were* presented as pre-unified, I would have to dismember it first in order to understand it, because I can only understand combinations that I make. I would have to dismember the object in order to have elements that I could then (re)-combine *myself*. Only then would the object's unity be *understood*.

Only a performed combination can illuminate the nature of an object's internal relations. Only synthesis can present relations to a unitary knower. Synthesis must be subject-performed because for a combination to be understood the knower itself must be the agent that spans, as it were, the "gaps between" the elements of the combination. Only performed combinations can be emulated in judgment, and the only combinations I can emulate in a conscious performance are ones that I myself have *already* carried out, consciously or not.

B130: I can resolve (via logical analysis) only what I have previously combined

The only kinds of combination-relation that I can understand are ones that I have *previously* combined. Kant explains what he means by reference to logical analysis. (Indeed, he says he calls the combining process "synthesis" precisely to call attention to the fact that he conceives of combination as that power of logical analysis in reverse.) Now, logical analysis is surely performed by the understanding. But how is it possible? I am able to divide logical complexes along logical lines, Kant says, only if I have *previously* logically combined them into logical complexes. For example, I can analyze the complex concept "red isosceles triangle" into just those three predicates only if I have

⁴⁸ "But combination does not lie in objects, and can by no means be borrowed from them by perception and thus be taken up only then into the understanding" [B134]. Also, at B153: "even if an intuition were already given in sensibility, the understanding cannot take it up *into itself*, in order-as it were-to combine the manifold of [what would then be] *its own* intuition."

previously constructed it out of them: “where the understanding has not beforehand combined anything, there it also cannot resolve anything, because only *through the understanding* could the power of presentation have been given something as combined” [B130].

The unity of understanding is *always* synthetic, even when what is understood is expressed in an analytic judgment—i.e., even when the “manifold” is simply the relation of a predicate concept to a complex subject concept that contains it. This is because analytic judgment is a performance involving a plurality of *consciousnesses*: “the consciousness of the one presentation [the subject-concept] can nonetheless, insofar as we are talking about the manifold, always be distinguished from the consciousness of the other presentation [the predicate-concept]” [B130 fn. 191]. Analytic judgment is a temporal performance: I think the subject and predicate at different times, and thus with different consciousnesses. Thus analytic judgment still requires a combination of consciousnesses—i.e., a synthesis.

Synthesis presupposes a unity. In § 16 Kant tells us that the source of this unity is the necessary unity of the synthesizing subject.

§16—ON THE ORIGINAL SYNTHETIC UNITY OF APPERCEPTION

B131: apperception means conscious perception

Kant inherits the term *apperception* from Leibniz. It means perception-with-consciousness. Apperception is distinguished from perception without consciousness, perception that is “nothing to me.” Like Leibniz, Kant accepts the possibility of presentations of the unconscious kind. Apperception is consciousness that is fully aware of its object, and with this arises its potential awareness of itself as subject. Henrich defines apperception as “The consciousness in which one knows that one can add the

thought of oneself as a thinking subject to each of one's thoughts" (Henrich, "Identity" 164).

on Leibnizian apperception

We can understand Leibniz's notion of awareness by comparing it to that of Descartes and Locke. For Descartes and Locke, it is impossible for a presentation to be unconscious—*every* datum is a mode of conscious awareness. Locke said that it is "impossible for anyone to perceive without perceiving that he does perceive" (Locke, *Human Understanding*, Book II, Ch. 27). Both hold that every presentation is essentially conscious.

But Leibniz allows for *unconscious* presentations—the so-called *petites perceptions*:

there are a thousand indications which make us think that there are at every moment an infinite number of perceptions in us, but without apperception and reflection, i.e., changes in the soul itself of which we are not conscious, because the impressions are either too slight and too great in number, or too even, so that they have nothing sufficiently distinguishing them from each other. (Leibniz, *New Essays* 47)

Since unconscious ideas are possible, the mere existence in me of a presentation (perturbation, modification of the subject) cannot be a sufficient condition of my awareness of it. There is a *further* condition—the presentation must be "apperceived" by me (i.e., reflexively grasped by the mind). In § 4 of the *Principles of Nature and Grace*, Leibniz says that "it is well to make a distinction between perception, which is the inner state of the monad representing external things, and *apperception*, which is consciousness or the reflective knowledge of this inner state itself and which is not given to all souls or to any soul all the time" (Leibniz, *Philosophical Papers* 637). A voice recorder has states

of perception, but not of apperception, in which internal states present something to a *subject* of perception.

Apperception is awareness that one is aware of a given presentation. Because I am aware of being a subject of perception, this being-a-subject itself becomes a presentation. I refer to this presentation of my own subjectivity by asserting “I think.” The *I think* is an assertion that has a referent, and this referent is my being-aware.

B132: analytic unity of apperception and synthetic unity of apperception

The *I think* must be *capable* of accompanying all my presentations. For otherwise something would be presented to me that could not be thought at all—which is equivalent to saying that the presentation either would be impossible, or at least would be nothing to me. Presentation that can be given prior to all thought is called *intuition*. Hence everything manifold in intuition has a necessary reference to the *I think* in the same subject in whom this manifold is found. [B131–32]

If I am aware of presentations *a*, *b*, and *c*, then I am also aware that *a*, *b*, and *c* all have one and the same subject. This is called the **analytic unity of apperception**, because it is the basis of analytic judgment. I can think many presentations under *red* only if these red presentations are one and all mine. My ability to subsume many instances under one universal rests on these instances’ all being presentations of one subject.

There must be only one subject of all my presentations. This means not only that I am aware *serially* of being the subject of *a*, and again being the subject of *b*, and again being the subject of *c*, but also that I must be *able* to be aware of them all-at-once *and together*. That is, I must be able to be aware of being the subject of *a* AND *b* AND *c*: “they surely must conform necessarily to the condition under which alone they *can* stand together in one universal self-consciousness” [B132]. This is called the **synthetic unity of apperception**.

All objects of intuition arise as a plurality but are objects of a unitary subject. So all the elements of a plurality have a “necessary reference to the *I think* in the same subject in whom this manifold is found.” The object is a plurality of passing pixels that is *thought* as being unified in various ways. The object is comprised of ways-of-plurality that are yet imagined (and thought of) as being nonetheless connected.

B132–3: universal relation of all (my) data to one subject of awareness entails their relation to each other

Until now, Kant has been treating the synthesis and unity of *presentations generally*—that is, both universals and intuitions. This is because his topic is synthesis, and synthesis is carried out by understanding, which is independent of intuition and so not limited by the latter’s conditions. At B133 Kant narrows the scope of his discussion to intuition “as such,” by which he means *any* kind of sensible intuition, not merely human intuition, which happens to be spatiotemporal. [B149] However, to facilitate my elucidation of Kant’s account, I will assume this condition in my discussion, simply in order to make it intelligible to humans who can only intuit sensations as being separated/connected spatially and temporally.

Sensibility gives me (a unitary knower) a plurality of point-moments in spatial and temporal separation. But a numerically identical subject can be aware of this plurality only if it can also recognize its identity at every point-moment. Each point-moment refers to the same subject. This entails that they must also be able to “stand together in one universal self-consciousness” [B132]. If the subject of a plurality of point-moments can think the same “I” at every point-moment, it must also be able to be conscious of the elements of this plurality *together and all-at-once*. For a unitary subject to be *conscious* of a plurality, it must be conscious of this plurality in its unity *as* a unity, and so must also be conscious of how the elements of this plurality are *interrelated*.

Sense-consciousnesses arrive as a plurality that is dispersed along the dimensions of spatial and temporal difference. Their belonging to one and the same subject is the same thing as one and the same subject *spanning across* these dimensions. And what is a spanning-across on the side of the subject is a combining-together on the side of the object. The act by which I realize my numerical identity across space and time *is the same as* the act of combining point-moments: the “identity of the apperception of a manifold given in intuition contains a synthesis of presentations, and is possible only through the consciousness of this synthesis” [B133].

To understand a plurality I must first present it as a plurality-in-unity, and this requires being aware of how the elements in the plurality are *related* to each other. Only combination can make this relation understandable. Understanding the relations of this togetherness, Kant says,

comes about not through my merely accompanying each presentation with consciousness, but through my *adding* one presentation to another and being conscious of their synthesis. Hence only because I can combine a manifold of given presentations in one consciousness, is it possible for me to present the *identity itself of the consciousness in these presentations*. [B133]

B134: act is prior to awareness and determines conscious synthesis

All the elements of a plurality can belong to *me* only if I am aware

that I unite them, or at least can unite them, in one self-consciousness. And although that thought itself is not yet the consciousness of the *synthesis* of the presentations, it still presupposes the possibility of that synthesis. I.e., only because I can comprise the manifold of the presentations in one consciousness, do I call them one and all *my* presentations. [B134]

The togetherness of marks in an object is a result of being aware that I *did* unite them or *can* unite them “in one consciousness.” An understandable combination is one that is self-performable, and a self-performable combination is one that *has been* self-

performed, consciously or unconsciously. Kant points out once again that the level of my awareness is variable—I might have synthesized the elements consciously, or I might only be aware that I *can* do so. In either case, the subject itself must be the source of this combination, and by means of its own act. *Act is prior to awareness*. This is why merely being aware of a plurality as *mine* “is tantamount to the thought” of my *past* synthesizing, even if I cannot recall having performed it. Although an act of synthesis *might not have been* carried out with full consciousness, it *can* be. Since I can emulate only what I already know, this proves that I have done it.

The acts of synthesis that we carry out automatically can also be performed intentionally. This must be done in order to make blind synthesis intelligible. For this reason, we should take seriously Kant’s explanation of how the conscious rendition is carried out. Even if this act is not identical with the act of blind synthesis, it is for us the only source of our understanding. When I consciously combine in order to understand (with consciousness), I am following a trail laid down by my own prior (albeit unconscious) combination. This prepared path was made for me by myself. Kant calls this prior act of path-making the **original unity of apperception**. [B135] By following the path of my prior act consciously, I learn about my own unconscious spontaneity and discover its rules. Kant will now show us, in the upcoming sections, how this intentional emulation of prior unconscious synthesis is to be carried out intentionally.

§17—THE PRINCIPLE OF THE SYNTHETIC UNITY OF APPERCEPTION IS THE SUPREME PRINCIPLE FOR ALL USE OF THE UNDERSTANDING

B136: unity is just as necessary for presenting as space and time are for receiving

We already know from the Transcendental Aesthetic that all sensible plurality is a priori conditioned by forms of space and time. Now, in the Transcendental Deduction, we

are told that all plurality for a unitary subject must be “subject to conditions of the original synthetic unity of apperception” [B136]. These are the two *conditions of presentation*, the two ways in which “presentation can make the object possible.” Just as the sensible plurality is subject to the forms of space and time by being given, it is also subject to the conditions of being an object for a unitary subject by being understandable—i.e., by being thinkable and amenable to truth-claiming. This condition is that “they must be capable of being *combined* in one consciousness” [B136]. Otherwise, the plurality would not have a unitary subject. A plurality can be a plurality for a unitary subject only by being synthesized by that subject.

B137: object-concept is really just the “unity of consciousness in their synthesis”

Now, an object is “that in whose concept the manifold of a given intuition is united” [B137]. An objective unity must be a unity-for-me, and this requires unity of consciousness at every point-moment, which in turn requires consciousness of their (previous or possible) synthesis. So the “concept” that unites the plurality of point-moments into unity of the object is really just *the unity of consciousness in their synthesis*.

B138: “I must draw it”

Kant calls synthesis a combining or an adding. When dealing with **logical objects** (objects that are complexes of universals), the identical act that unifies is logical combination. When dealing with **sensible objects**, all of which are bodies, and hence spatially extended, the identical act that unifies is **line-drawing**:

the mere form of outer sensible intuition, i.e., space, is as yet no cognition at all; it provides only the manifold of a priori intuition for a possible cognition. Rather, in order to cognize something or other—e.g., a line—in space, I must draw it; and hence I must bring about synthetically a determinate combination of the given manifold, so that the unity of this act is at the same time the unity of

consciousness (in the concept of a line), and so that an object (a determinate space) is thereby first cognized. [B137–38]

The plurality even of empty or pure space can become a unity for me only when my common act of synthesis spans throughout space in this way. Space as form of intuition is no unity; rather, “it provides only the manifold of a priori intuition for a possible cognition”—i.e., it merely provides the separability conditions for a plurality. As such, the form of space is nothing for *me*, a unity. For space to be a unity for me, I must fill it with my activity. This means *positing* images in the imagination—in this case, a series of points. The only points that can be mine are ones that I have *posited*, ones containing evidence of my *act*. I appropriate a point of space by filling it in the imagination. I then combine these appropriations by positing *again* along the dimension of difference offered by the form I am unifying. So I posit in a *different* form-way, in this case, in a different *position*. But this latter positing is not merely placed in a different position, it must also *act* its way there, and in a way that brings the previous point “along with” the new one. I act my way from point to point in a way that retains the past ones. Not only positing, but also combination, must stem from a unitary actor. This acting my way across different positions, Kant is saying, must be rendered as movement. Movement is the fundamental combinatory act for synthesizing space.

A plurality becomes united into apperception-enabling activity only when its elements are *added* together. Adding is how combination becomes activity. Logical adding is familiar from Aristotelian logic—an identical agent repeatedly combines predicates through alternating use of the logical AND. Now Kant tells us how such adding takes place *in sensibility*, and that this adding is necessary even for the intuition of empty space. If I am aware of space, my awareness is at every point. If my awareness is at a point, then I either have or must be able to posit this point as a content in the

imagination, since awareness of a particular as *P* means being able to emulate it as a self-made image. But space is plural and must be apprehended sequentially. In order to apprehend *into* unity, my act cannot be one of positing a series of adjacent points, but one of by *continual* point-positing, i.e., through line-drawing.

As we will see in more detail later, Kant conceives the combining of sensible (physical) objects in terms of line-drawing performed as the act of *moving an identical point* through space (and over time). When synthesizing space (or time), the identity of the agent rests not merely on a repeated (identical) act of conjunction, but in a *continual positing of a point intended as continually self-identical*. Continual identity is posited *through* a plurality that is now produced, not by positing different universals, but by *moving*. Motion generates plurality by act—it generates the plurality while unifying it in the activity of this generation.

Imaginary spatiotemporal *traversing* is thus the sensible analog of logical combination, which is *time-order irrelevant*. I (re)-combine the marks of a logical object in any order, but succession is determinately ordered and one-dimensional when I draw a line.

§18—WHAT OBJECTIVE UNITY OF SELF-CONSCIOUSNESS IS

B139: objective validity as the fixed order of time

The *transcendental* unity of apperception is the unity whereby everything manifold given in an intuition is united in a concept of the object. Hence this unity is called *objective*, and must be distinguished from *subjective* unity of consciousness, which is a *determination of inner sense* whereby that manifold of intuition for such [objective] combination is given empirically. [B139]

This is the second time Kant has mentioned concepts. As he did at B137, he again identifies the *concept* with the mere *unity* of the object. This concept-object is the end product of synthesis—the final unity that is produced on the object-side of consciousness.

This section is not easy to interpret. I suggest using the opening sentence as a guide. Kant has already said that the unity of apperception should be called the “transcendental unity” of apperception. Recall that *transcendental* means *explanatory of accepted a priori knowledge*. Kant is now moving into an explanation of objectivity. We have noted that intersubjectivity is especially problematic under representationalism, and that Kant tackles it by reducing intersubjectivity to epistemic necessity (necessary truth), and epistemic necessity in turn to genetic necessity (innateness). Epistemic necessity is what we acquire in the a priori sciences—logic, mathematics, and mathematical physics. Unitary apperception is called *transcendental* because Kant intends to use it to explain the possibility of these sciences. The necessary truths of these sciences are possible because their respective objects have as their unity the very ways-of-combination that are necessary in order for identical consciousness to span over the ways-of-separation that must be overcome in order for understanding (judgment) to occur: spatial, temporal, and qualitative.

So Kant is here arguing that the *objective* (intersubjective, necessary) unity of objects has the unity of apperception as its transcendental basis, which he now calls the *objective* unity of apperception. And Kant explains this objective kind of unity by contrasting it to the subjective (empirical, contingent) kind.

The (merely) subjective unity of consciousness is “a *determination of inner sense* whereby that manifold of intuition for such [objective] combination is given empirically.” By “determination” here Kant means my ability to be conscious “*empirically* of the manifold as simultaneous or as sequential” [B139].

Kant seems to be talking about the spatial and temporal arrangement of point-data. From the Humean perspective, the spatial and temporal situation of point-data is taken solely in terms of their *contents*. From the fortuitous fact that spatial and temporal

relations of point-data are regular, I am able to forge an **empirical unity of apperception**—a nexus of content-based associations. But, as Kant shows in mind-numbing detail in the A-Deduction, this associative unity rests on another—i.e., on the fact that there is a stable, universal, intersubjective framework of space and time in the first place. The accidental *associative* links of the Humean kind occur within, and thus *depend* on, the framework of objective time: “the pure form of intuition in time, merely as intuition as such containing a given manifold, is subject to the original unity of consciousness. It is subject to that unity solely through the necessary reference of the manifold of intuition to the one [self], i.e., to the *I think*” [B140]. My internal time-line is fixed, no matter what its contents. Only this “original” unity of consciousness is “valid objectively.”

The contingent forces of association reproduce contents based on the contingent forces of association, but they do so in the fixed order of time based on the contingent forces of association. Kant gives the example of word-association. The time-series of contents that follow from association-based reproduction will vary from person to person—i.e., it has only *subjective validity*. But we are all aware that our subjective series are embedded in one shared intersubjective time. The empirical order of time is subjective, but “the pure form of intuition in time, merely as intuition as such containing a given manifold, is subject to the original unity consciousness” [B140].

Take the psychoanalytic technique of free association. Even though the series of imaginary contents is determined by accidental force of my biography, I am still aware that the accidental series of images in my inner sense occurs within the context of an objective or “pure” time-order, which I access through outer sense. I am aware that I am a body, sitting in a room, where objects such as physical clocks exist. This is the outer world that serves as the standard that lets me order my image-stream in objective time.

Even while daydreaming, I am still able to refer the order of my imaginings to objective time—I *know* that I thought *a* at 1700 and *b* at 1800.⁴⁹

§19—THE LOGICAL FORM OF ALL JUDGMENTS CONSISTS IN THE OBJECTIVE UNITY OF APPERCEPTION OF THE CONCEPTS CONTAINED IN THEM

B140: Kant cannot settle for the traditional theory of judgment as a combination of universals

“I have never been able to settle for the explication that logicians give of a judgment as such. A judgment, they say, is the presentation of a relation between two concepts” [B140]. For Kant, the function of the copula *can* be taken as a relation between universals, but only in the case of analytic judgments about *logical* objects (although these may eventually refer to sensible objects, as they often do).

The problem with this definition is that it does not explain the possibility of ostensive judgment. Ostensive judgment aims directly at sensibility and asserts truth of a different kind—i.e., truth about physical objects. The recurring problem of the First *Critique* is how such physical cognition can be possible, given that its sensible matter presents no such thing, but only passing arrays of pixels. The answer is that the same structure of judgment which *recognizes* the object also serves as the template for constructing it: “The same function that gives unity to the various presentations *in a judgment* also gives unity to the mere synthesis of various presentations *in an intuition*” [A79/B104]. Physical objects are forced into existence *by* the act of asserting truth through the structure of judgment towards the passing plurality of sensibility. The physical object, which is the referent of possible sensible truth, is forced into existence

⁴⁹ This ability to determine inner sense by reference to outer sense is itself a “transcendental condition.” I can *only* determine the order of my inner contents by referring them to outer sense. This is precisely the point of the Refutation of Idealism, and the motivating force behind Kant’s revision of the *Critique*.

because “This S is P” contains functions of unity (ways of combination) all of which have the *I think* at their apex.

This is the key to understanding Kant’s theory of objective construction. Physical objects are passing pixel arrays whose elements (point-moments) have been combined in certain ways. *These are the ways of unity thought by the judgment-forms*. Thus judgment provides the template according to which our understanding automatically combines point-moments into combinations that *can* be consciously recognized. My ability to *recognize* this combination and emulate it consciously in judgment means that I must be able to emulate its constituent acts of combining with full consciousness. Doing this means traversing these ways of plurality-in-unity in the imagination, i.e., through *line-drawing*. These unities are objective because they are necessary—both the forms and the unity of apperception are necessary. The judgment-forms are rules for combining pluralities by traversing them (or by generating change internally so as to produce them from a single act), and the principle of apperception means that *fully conscious* traversing must be possible.

B141: the “little relational word is”

In the second paragraph, Kant returns to the distinction between the subjective (contingent) and objective (necessary and thereby intersubjective) unity of consciousness. By subjective unity, Kant means the strictly empirical or content-based unity of association, or reproduction. Contents are linked by force of habit. The occasion of one content spurs the imagination to reproduce another. This is the unity that Kant calls subjective, empirical, associative, or reproductive. Kant’s interest is in explaining the *other* kind of unity—the unity that is a transcendental or explanatory basis for a priori knowledge, which we do in fact have. We are now being told that this basis is twofold.

The origin or force that drives this unity is the original unity of apperception. This has been the topic of the B-Deduction up to now. But this unification is not featureless—it also has a particular structure, and that is the *structure of judgment*. Looking at judgment with an eye to explaining intersubjective agreement, Kant says,

I then find that a judgment is nothing but a way of bringing given cognitions to the objective unity of apperception. This is what the little relational word *is* in judgments intends [to indicate], in order to distinguish the objective unity of given presentations from the subjective one. For this word indicates the reference of the presentations to original apperception and its *necessary* unity. [B141–42]

Hence the title of this section. Kant is *identifying* the structure of apperception (the indirect self-awareness that follows simply from being aware of something) with the structure of judgment, “S is P.”

B142: “It, the body, is heavy”

Kant gives an example. According to the laws of association,

all I could say is: When I support a body, then I feel a pressure of heaviness. I could not say: It, the body, is heavy—which amounts to saying that these two presentations are not merely together in perception (no matter how often repeated), but are combined in the object, i.e., combined independently of what the subject’s state is. [B142]

This is how Kant explains the relation between objective validity and judgment. A subjectively valid relationship is time-irrelevant. Image *a* comes to mind, and then image *b* follows due to the force of association. This is a sequence, but it is not *real*. A real sequence is one that is lawful and holds “independently of what the subject’s state is.” The law is a time–state law: a law that determines the content at every point-moment as a function of time. This time–state law is the referent of the copula in ostensive judgment.

§20—ALL SENSIBLE INTUITIONS ARE SUBJECT TO THE CATEGORIES, WHICH ARE CONDITIONS UNDER WHICH ALONE THEIR MANIFOLD CAN COME TOGETHER IN ONE CONSCIOUSNESS

B143: Kant reviews his main points

Kant now provides a summary of his core argument and shows how it proves the a priori objective reference of the categories.

§ 17: Sensibility delivers a plurality of sense-consciousnesses. But epistemic consciousness is unitary. Only the *act of combining*, carried out by the understanding subject, can solve this problem.

Combination is carried out by the understanding. The understanding knows facts by resolving complexes into elements and then (re)-combining these elements in the structure of judgment. It beholds an object, recognizes the object as a fact, and then produces this fact in language, as a relation between concepts, e.g., *This table is brown*. It is this *productive* or *active* mode that manifests the self-as-agency, and this is the referent of the *I think*.

The understanding knows facts only because it knows objects as complexes—it knows the inner nature of how these elements are interrelated. This is possible only by adding these elements together itself, i.e., by *providing* this inner relation. And so analysis presupposes synthesis. I can undo links and separate elements only if the complexes I am analyzing were assembled, by me, from the elements I can now extract.

§ 19: Apperception is the self-awareness that arises from knowing. When I claim truth, I am a subject that carries out an act of comparison—I assert a proposition (“This S is P”) and intend it as identical with some real fact. “This S is P” is true when some S that I can intend in perception is P. Self-awareness arises, then, when the self acts as the agent of knowledge—I *produce* the “This S is P” and test it against some real fact. The inner nature of the object, of truth, is knowingly self-made via the judgment-forms.

§ 13: The categories are “nothing but precisely these functions of judging insofar as the manifold of a given intuition is determined in regard to them” [B143]. The categories are universals that I abstract from the essential, intra-relational features of the GPO (generic physical object).

Therefore, “the manifold in a given intuition is subject necessarily to the categories” [B143].

§21 — COMMENT

B144: the Transcendental Deduction has only “begun”; outline of what comes next

Kant says that he has now shown that the rules for presenting an object of knowledge *in thought* (in judgment), which are therefore necessary for bringing a subject into existence (since the subject knows only by asserting “is P”), are just as necessary for the presentation of an object as the forms of space and time. The presence of the subject is possible *only* through subject-manifesting activity, and the *unity* of apperception is possible only when act is unifying. The contents of the object are differentiated a priori by space and time, and then unified a priori by the judgment-forms.

The infamously puzzling remark in this section is Kant’s claim that he has “made the beginning of a *deduction* of the pure concepts of understanding” [B144].” By all appearances, § 20 should have been the *completion* of the Transcendental Deduction. But, he says, this deduction is not yet complete because he has been describing (as is proper) understanding in abstraction from our particular way of intuiting. Since understanding is independent of sensibility, the proper way to proceed is to describe synthesis in the most general terms. Only later, in § 26, will Kant restrict his description to synthesis of the sensible kind. He also plans to show that synthesis is necessary, not only for *particular* unities (i.e., physical objects), but even for the very fields of space and

time as such. Space and time *themselves* are not presentations for consciousness until after synthesis has occurred. Then, he says, the Transcendental Deduction will finally be complete.

§22—A CATEGORY CANNOT BE USED FOR COGNIZING THINGS EXCEPT WHEN IT IS APPLIED TO OBJECTS OF EXPERIENCE

Kant here distinguishes *thinking* from *cognizing*. Thinking is the power of assertion, and so flows from the subject of knowledge. For example, when I think the subject-position I intend a certain objective combination. But this object need not be given—i.e., intuited as an immediate and particular presence. This would be “a thought without any object, and no cognition at all of any thing whatsoever would be possible by means of it” [B146].

Cognizing, on the other hand, “involves two components: first, the concept (the category), through which an object as such is thought; and second, the intuition, through which the object is given” [B146]. When I cognize, I think an object *through* a pure concept (judgment-form) and *towards* an intuition.

Finally, Kant gets to his point: cognition by definition is cognition of *reality*. We can have knowledge without reality, but not cognition. Examples of mere knowledge are analytic and mathematical truths. In these I assert a truth whose object I construct without dependence on reality. In analytic judgment, I know something about a logical object that I make through acts of the understanding alone. In mathematical judgment, I know something about a mathematical object that I invent in pure intuition, i.e., in intuition empty of reality. In these ways I assert truth (and justify correspondence) without having

to relate it here and now to something real, something that perturbs me extra-volitionally. This, Kant says, is required in cognition.⁵⁰

The same holds for mathematical truths. These always refer to intuitions, but not to anything received. In order to know them, I do need to make intuitions (images). I need to posit points, draw lines, and measure distances (also by drawing lines). And I need to posit marks of some kind when I count. But mathematical concepts refer not to images, but the pure framework that contains them. They refer to facts about *mere* space, and magnitudes made of *mere* time. “Consequently all mathematical concepts are, by themselves, no cognitions” [B147].

In summary, cognition has reference to reality because of its passivity. It is the constant stimulation of sensation by some real unknown that is basis of the reality of our shared physical world. Imaginings are the archetype of what we mean by private-world experiences. And so Kant defines experience as empirical cognition.

§23

Here is where Kant makes room for our necessary illusions: “Space and time, as conditions for the possibility as to how objects can be given to us, hold no further than for objects of the senses, and hence hold for objects of experience only” [B148]. For Kant, there is a world beyond that of experience. We are compelled by our constant effort towards judgment to believe that its referents apply independently of our intuition. These may or may not exist in the same way as physical objects, whose matter is the reality of sensation. Such **transcendent objects** are not objects of experience because they are acts of thought without content, and thus without objects. They are, in other words, *ways of*

⁵⁰ However, we should note that all concepts gain their sense from intuitions. To verify significance, I must schematize in intuition. Say I make the analytic judgment that “All red triangles are red.” This is true independently of intuition. However, its sense is not. I gained the sense of *red* from sensibility.

assertion that intend something that is never given. Only what is in space and time is given, and this given is sensation.

What results from applying judgment beyond our intuition is the absurdity of asserting what *would* be categories (which are abstracted from *empirical* combining by the judgment-forms) towards an object that is absent. For example, if I “apply” the predicate-position towards what lies beyond possible intuition, what results is the awareness “that the object has as a property nothing belonging to sensible intuition” [B149]. Try as I might, I cannot imagine what this could be. I assert an object-expecting intentionality but remain empty of the satisfaction of meeting my intended referent. There may be objects “there” (though not spatially), but they are not objects to which my categories can apply. There is thus no encounter, no experience, and even no knowledge. (Unlike the empirically empty truths of analytic judgments and mathematics, the intuitionally empty principles stemming from objectless asserting through the judgment-forms cannot even be true.)

§24—ON APPLYING THE CATEGORIES TO OBJECTS OF THE SENSES AS SUCH

How can empty forms of judgment acquire objective significance and reality?

We have seen that pure concepts rest in understanding and refer originally to apperception, whose unity and genesis they serve, and so have no inherent limitation on their application.⁵¹ But Kant’s goal is to establish knowledge of *reality*, and reality is what comes from otherness. Our capacity for being stimulated by otherness is sensibility. So Kant has been considering how the categories function in relation “to objects of

⁵¹ See § 23. Also, recall B130: “all combination is an act of understanding—whether or not we become conscious of such combination; whether it is a combination of the manifold of intuition or of the manifold of various concepts; and whether, in the case of intuition, it is a combination of sensible or of nonsensible intuition.”

intuition as such, whether this intuition is similar to ours or not, as long as it is sensible rather than intellectual” [B148]. By sensible intuition “as such” Kant means any mode of passively accessing reality as a plurality of particulars.

But a concept can have determinate meaning and significance only if it has *objective* (and not merely intended or “transcendent”) reality. For humans, reality comes by way of intuition that is preconditioned by the forms of space and time. However, because the understanding is independent of all sensibility, I can *attempt* to apply pure concepts to other kinds of sensible-intuitive objects, whatever these may be. But attempting this will leave me objectless, for in such a case “the pure concepts of understanding are then mere forms of thought, without objective reality” [B148].

Pure concepts are originally mere **forms of thought**, for they must function independently of intuition. They seek ways of unifying plurality, even if no plurality is given. In the generic theory of synthesis, transcendental unity of apperception says that I must be able to be aware of myself (as subject) across *all* the ways-of-plurality that I can think as combinations and thereby resolve—logical, spatial, temporal, and transcendent. Each domain of application is a dimension of difference that contains its own way-of-plurality, and my unitary awareness must span across it and unify it into the structure of judgment.

Up to now, Kant has described the workings of understanding in the most generic terms, in order to make good on its independence from sensibility. We are now told that the activity of understanding taken alone is called **intellectual synthesis**. In intellectual synthesis, the combination of the manifold of *presentations generally* “referred merely to the unity of apperception, and was thereby the basis for the possibility of a priori cognition insofar as such cognition rests on the understanding; and hence this synthesis was not just transcendental but was also purely intellectual only” [B150]. The pure

concepts are rules for “thinking” in general—rules for asserting “S is P” about anything. Their job is to think *any* plurality into the final combination unity of apperception.

The pure concepts are rules for enabling unitary apperception. Epistemic consciousness arises *in the act* of asserting “S is P.” This effects a synthesis of the most abstract kind. The synthesis of understanding into the unity of the copula is the “highest” synthesis of understanding since it refers “merely to the unity of apperception” [B150]. It is what finally brings things to the unity that is essential to the unitary subject. There is *one* subject of a sensible *plurality*. This plurality comes into *conformity* with the unitary subject by being synthesized so as to present an *S* and a *P*, which are in turn combined via the “is” into a final unity—compatible with the unitary subject that asserts “I think that S is P.”

The pure concepts must have objective reality if they are to have reference and significance. How does this come about? How can a *pure* concept, one that applies in *all* knowledge precisely because it is *not* empirically derived, have objective reality? We know that the pure concepts are functions of unity in judgment—rules of synthesis necessary for understanding, awareness, and thus apperception. Their mode of application is *constitutive*: a pure concept enters into objects *by means of synthesis*. But how exactly does this happen?

The pure concepts, taken in isolation from this or that kind of possible sensible intuition, are nothing more than functions of **intellectual synthesis**. To grasp their significance, we must specify the *particular* nature of the plurality they intend to synthesize. Any plurality is pluralized in a certain *way*. Only after specifying the way-of-separation that the pure concept is out to combine (spatial, temporal, or qualitative) can we finally determine (1) how it can apply to that kind of sensible intuition (i.e., have objective reality), and (2) what the pure concept *means*.

Kant now turns to the particular (and a priori) form of separation that human sensible intuition actually provides. My sensible subjectivity is pluralized at its foundation, which is inner sense, and this plurality has the a priori form of time: “there lies at the basis in us a priori a certain form of sensible intuition, a form that is based on the receptivity of our capacity to present (i.e., based on our sensibility)” [B150]. Our innate rules of apperceiving, the pure concepts, which are the judgment-forms guiding synthesis, gain objective reality not by applying to sense contents as such, but to the *form* that receives them. Unlike the reality that produces the contents of sensibility, the *form* of sensibility is innate. Pure concepts apply to the pure framework within which the plurality of my sense-contents is contained: “Hence the understanding (as spontaneity) can, by means of the manifold of given presentations, determine inner sense in accordance with the synthetic unity of apperception” [B150]. Kant calls this, the application of the judgment-forms to sensible reality by employing them as rules that “determine inner sense,” **figurative synthesis**. It is by means of figurative synthesis that the understanding becomes able to “determine inner sense in accordance with the synthetic unity of apperception; and thus it can think synthetic unity of the apperception of the manifold of a priori sensible intuition” [B150]. Figurative synthesis is what the apperception-enabling act of understanding is called when it takes its object from sensibility.

Why *inner* sense?

Why would Kant choose inner sense as the domain of synthesis? Recall that in the Transcendental Aesthetic, Kant prioritized time over space with regard to *containment*. Time is the form of *all* intuition, inner and outer. [A54/B60]⁵² The temporal manifold

⁵² It is important to note that with regard to *intelligibility and significance*, this situation is reversed—space takes priority over time. *This is a key premise of my thesis*, and also the key difference between the two

contains *both* the manifold of space *and* the manifold of passing away. This is a *time-slice* theory of sensibility—spatial plurality is embedded *within* temporal plurality as a slice in a continuum. It is the plurality of passing away that, because it is the most inclusive plurality, must be chosen as the primary way-of-separation that must be overcome by figurative synthesis. There is another reason why time is prior to space. Formal space must be apprehended before it can become a unity. Constructing space is a time-taking activity. I construct space point-by-point, and each point occupies a moment of inner sense.

When the pure concepts apply to the plurality of inner sense as rules of figurative synthesis, they do so by taking hold of the *form* of inner sense. What is combined, then, are not moments as *contents*, but moments as *positions* in the framework of time. The pure concepts combine the matter of inner sense *through its formal framework*. The pre-divided framework of time is the scaffolding of which provides the handles that the pure concepts need to operate. Temporal separation is the Kantian equivalent of Aristotle's prime matter. This is what must be combined in order for the pure concepts to work their way into objects of human knowledge and thereby acquire objective reality.

Now comes the key to my thesis. Kant next explains how it is that the subject's productive activity, the ontological basis of the subject as *appetitus* (and the referent of the *I think* that must always be possible), becomes constitutive of objects.

When the figurative synthesis “concerns merely the original synthetic unity of apperception,” it is called **transcendental synthesis of imagination** to distinguish it from the merely intellectual combination of the understanding. [B151] **Imagination** in general is defined as “the power of presenting an object in intuition even without the object's

editions of the *Critique*. Kant refutes his detractors' accusations of idealism by pointing out repeatedly that, while the *form* of inner sense (and therewith the self) is time, time cannot be presented except in dependence on *outer* intuitions. This will be discussed in detail shortly.

being present” [B151]. The imagination is the ability of the active subject to stimulate its own passive power to produce faint intuitions, called *images*. Since imagination is a power of making intuitions, Kant says the imagination “belongs to *sensibility*” [B151]. But while sensation is other-affected and delivers an uncombined plurality of sense-atoms imagination is self-directed and can be used not only to posit self-made contents but also to *combine* them. Sense is “merely determinable,” while imagination “is an exercise of spontaneity, which is determinative” [B151]. The important result is that figurative synthesis “can a priori determine sense in terms of its form in accordance with the unity of apperception” [B152].

I know the form of inner sense a priori. And the spontaneity that forces the imagination to carry out acts of combination has my understanding as its subject—understanding has a priori rules, which are those of logic. This is how my conditions of presentation determine the object. This is the work of what Kant calls **productive imagination**. In reproductive imagination, such as goes on when I think *red* and then produce an image of it, I am making a direct copy of a *given* content. What originates as given can only be *re*-produced. But the features of the generic physical object (the categories) are not even originally presented as contents. My imagination is not imitating anything given, but rather inventing for the first time.

The unity of apperception is what follows from the act of positing “S is P” as true of an object. If I see a green chair, I can become aware of what I am seeing. I am aware of “it” by knowing “what” “it” is. I claim the “what” with the force of asserting its identity with the “it.” The “it” is an *instance* of the “what.” Being aware is being aware *of* something, and to be aware of *something* is to know its “what” or kind—something general that can be found in multiple “it”s. Knowing a kind means knowing the rule (of

combination) I should follow in order to produce a given “it.” The rule is what I contribute when I assert “S is P” about some *S* that is *P*.

Kant is now saying that just as my empirical awareness is realized by advancing a rule and claiming that the “it” before me falls under the rule by being identical to an image that I can produce by following the rule, so also my awareness of the generic physical object is produced is realized by advancing the rules that comprise the generic “This (S) is P.” But more than this, the rules that I advance are the *same* rules that produce the object itself—rules that force my productive imagination to invent the “it” from scratch. They are not rules that are *discovered* by comparing *given* instances through the process of reflection. They are my own rules for thinking *any* object of apperception, that is, any “it” that can be a “what.” But when these rules apply specifically to human sensible intuition, they manifest as ways of combining point-moments. These rules are originally the judgment-forms—subject, predicate, copula, quantifying the predicate wholly or partially, and affirming or denying the act of predication. These judgment-forms, Kant says, are the “rules” that produce an object that conforms to “S is P.” (Explaining exactly how this is done must wait until we discuss the Schematism.) Kant’s thesis is that the judgment-forms are rules that guide the imaginary ways of combination that, together, produce a cognition of the generic physical object.

To be aware is to advance an imitation of the object of awareness. I see a red datum, I am aware of it, so I think “red,” which is a rule for producing a matching imaginary instance of red, which happens to be an image of red. I posit “red” in response to a given red datum, and so make a possible *I think* actual. Understanding is the act of positing the kind, or rule, of an object. But there is a preliminary act of positing that is not the positing of the rule over a content, but the positing of the rule over a way of connection. I can imagine that two points are connected across space and through time.

Apperception makes a difference here as well. I must not only add things together, I must also be *conscious* of this adding. This must be possible, for this is how the adding becomes significant for consciousness. Remember that automatic synthesis is blind—carried out unconsciously, albeit by me, by understanding, and under the unity of my identical activity. But the meaning of this act does not exist for apperception except by intending it, so that activity occurs again, but this time under conscious direction and according to a rule. Automatic synthesis connects point-moments in a way that we can never know, but conscious synthesis imitates it, as we have seen, by drawing a line.

Line-drawing is a generic activity that can accommodate various *modi operandi*. That is, it can be carried out under different kinds of intent. I can intend my act to construct a line as part of a geometrical figure, as a tick-mark in the service of counting, as part of the outline of an empirical concept (Kant's example from the Schematism is a dog), as a way of measuring distance. But it can also be used to measure the passage of inner sense. I can “clock” the passing of my corporeal sensations by drawing at a constant speed. It can be used to present time itself, to give “before” and “after” real presence in intuition as, say, *left* and *right*, and thus as a visible *framework* for ordering actual events. It can also be used to construct a continuum for expressing the “intensity” (value) of a given quantified property, such as hue, mass, frequency, velocity, position, etc.

The figurative synthesis of line-drawing is *transcendental* when I carry it out under the intent *of a judgment-form*. The relations connected by the pure concepts, Kant says, are those of the pure manifold of intuition—space and time. Combining point-moments can only be accomplished *with consciousness* by doing so intentionally. And this can only be accomplished by drawing a line in the imagination.

What apperception refers to is its own unity, expressed as synthesis, under the guidance of the structure of judgment. Judgment is the means of awareness and the

vehicle of my epistemic subjectivity. The *I think* can accompany my awareness only when I am aware in a way that conforms to the conditions of asserting “This (S) is P.” Actually asserting this means doing so intentionally, with consciousness. Assertion is a production of an imitation of the object, and that imitation is judgment. So I must now synthesize in a premeditated way. For example, “This (S)” demands an intuition, which must be extended in space, i.e., a *body*. A body is a trans-positional *unity* of point-data. No such unity is given, so I make it in the imagination. I sweep along the point-data before me and unify them under the unity of a line, which is *one* because one actor draws it, and I have immediate access to this actor and its unity because I am identical with it. Similarly for the other kinds of combination demanded by judgment—substance, property, quality, and causality. Each judgment-form refers to a certain way-of-imagining the connecting of point-moments through the generic act of line-drawing.

So there are *two* syntheses at work. The first is intellectual synthesis, which determines the combination of universals in non-ostensive judgment, such as “Some leaves are green.” The second is figurative synthesis, which determines the combination of point-moments comprising the manifold of pure intuition. What the two have in common is that both bring their respective cognitions to the unity of apperception.

Judgment is a mechanism of combination containing essential functions of unity, which are the pure concepts. The pure concepts cannot find their way into the matter of sensibility, which are by definition received from otherness, but they can fill the “gaps” between them. Each pure concept is a judgment-form in the role of a *rule of synthesis*, a rule for determining how I must preconceive and intend my activity of line-drawing. These judgment-forms already serve as rules in general logic. The subject refers to a logical object, a logical combination of universals, that is a condition of true predication. A true predicate is a universal that is a component of this combination; the copula

reattaches this component after I intend it in isolation from the whole that contains it. The judgment-forms also include the rules of immediate inference: all, some, affirmation, and denial. These are essential because the telos of reason is a unified web of judgments and a hierarchy of universals in logical subordination. In general logic, the proposition is subordinate to the possibility of syllogistic inference, because it is only through inference that the unity of reason can expand.

But when facing the forms of space and time, and their plurality of passing point-data, these rules are still judgment-forms, but they serve a very different function. These are the rules described by transcendental logic. *Transcendental logic is the logic of figurative synthesis*. Combination under intellectual synthesis automatically brings the logical object under the rules of general logic. But combination under figurative synthesis automatically brings the sensible object under *the categories*—the universals that are abstracted from the imaginary unities whose rules are the judgment-forms.

Figurative synthesis is “an action of the understanding upon sensibility, and is the understanding’s first application (and at the same time the basis of all its other applications) to objects of the intuition that is possible for us” [B152]. These “other applications” are applications of universals—empirical, mathematical, and pure.⁵³ There can be no application of the *P* in “is P” unless a referent is presented for “This (S) is P.” This referent is the product of figurative synthesis—acts of combining point-moments done in the service of constructing *some* sensible object of apperception before empirical

⁵³ Applying pure concepts as universals, or categories, is a secondary use because I am then applying a universal to something that has already been synthesized. For example, the rule of the subject-position *in* synthesis is to *produce* the kind of imaginary unity that I subsequently abstract as *body* and *substance*. But doing this is different from my *application* of these universals *to* something that has already been synthesized. There must be a generic physical object before I can apply predicates to it. When applying categories as predicates, I must check for homogeneity, and I apply the predicate with an attitude of recognition rather than construction.

kinds can be predicated. This primordial sensible object is the **generic physical object**, and its essential predicates are the **categories**.

SPECIAL SECTION ON LINE-DRAWING

Just before B153, Kant marks off a special section that begins by noting that figurative synthesis is necessary even for the cognition of my (inner) self. The content of my self is just all the sensible contents of consciousness—all spatial points, which are in me as “outer” contents but in my stream nonetheless, and all the sensations of my body, including images in the mind; in other words, everything that is not a concept. When I cognize myself as a reality, *this* is what I am made of. But this state-filled self is a cognizable object, and an object of knowledge via judgment, and therefore subject to synthesis by the judgment-forms. The basis of this synthesis is my agency, my self as *appetitus*: “synthesis is nothing but the unity of the understanding’s act: the act of which the understanding is conscious as an act even apart from sensibility” [B153].

There are thus two sources of selfhood—sensation and act (spontaneity). About the latter self we only know its character of being-active. About the former we acquire everything that we know of ourselves besides our nature as being-active. These are separate. Spontaneous understanding is geared entirely for apperception. As we have seen this is shown by the fact that it applies to “the manifold of *intuitions as such*” [B154]. But inner sense also contains the form of temporal separation. Therefore, even my own inner self-cognition requires figurative synthesis.

So understanding has a dual allegiance. On one hand, it acts for the sake of an apperception that arises as “S is P.” For the pure concepts to be real, their functions of combination must be realized in the domain being combined. But act can be intuitive only through image-making, i.e., by positing something, at least a point (and *all* images,

being extended, are constructed piecemeal out of points), in imaginary intuition. This is because reality is extra-volitional stimulation of sensibility, and sensibility comes *through* the forms of intuition, space and time. Act has to conform to the way-of-separation that it intends to combine, and this conformity is realized by *making elements in it*. Through act I stimulate my own intuition as *imagination*, just as noumenal reality stimulates my intuition as *sensation*. My ability to determine intuition *begins* with my ability to act on it at all, and this requires being able to provide the content of a formal *element* (a point-moment). A point-moment can permit my act of combining it with others only if it is vulnerable to my act in the first place. *I can only combine point-moments that I can myself change by means of positing them as imaginary points*. I can control the form of figurative synthesis only if I can also posit its *matter*. What remains after I draw a line might be invisible, but the act itself must be carried out with a content—a content that I provide. It must be possible for my power to accompany every point-moment that I am aware of.

By positing the simple element of intuition, the form-to-be-combined is revealed along with it. As an illustration, we might imagine a familiar framework of separation, such as a honeycomb or ice-cube tray. Imagine that it is invisible—it is not yet an object for a unitary apperception. Nothing has been apprehended. In order to apprehend, I must make the framework visible. I could, for example, spray it with a colored liquid, or blow smoke at it. When I act by using the force of imagination to fill the framework of intuition, I must accommodate the form of the intuiting realm. In this way, I make intelligible its pluralizing structure.

Spontaneity and sensibility are opposites in Kant's system. Their interface is provided by synthesis. Spontaneity is the power of my identical act to produce imaginary contents and combine them. Understanding occurs only through judgment, which is itself

a combinatory structure. Knowing is predication, which entails knowing how to use a concept as a rule for making an instance identical to the presentation in the subject-position. But pure understanding is the ability to cognize a generic physical object, whose structure is that of ostensive judgment, whose functions of unity are necessary conditions of knowing anything at all. Sensible cognition therefore requires that the passing plurality of point-moments be combined according to the structure of judgment. Sensible reality conforms to the unity and structure of judgment. But sensibility does not just present just any pluralities; it contains a priori forms of pluralization. So synthesis by spontaneity must also do some conforming, for it has to accommodate the way-of-separation that it must overcome when being applied to sensibility. The understanding combines pluralities into unity, but it must conform to the way-of-separation that typifies any given plurality.

The understanding subject is a creator, and I conform to the form that governs my passivity. But creating must encompass not only the elements but also their plurality. *And so I must create the way-of-separation as well.* Only this can generate the plurality as a unity. Combination of multiple elements given through a way-of-separation must be carried out by internally producing this way-of-separation *as a way of change*. The way-of-separation must be actively produced as the way in which *I conduct this change*. To carry out synthesis *consciously*, I must *will* the change from element to element. Only by doing this can I *own* the medium of separation. Kant calls this internal production of objective separation *adding*—the old is retained and the new comes into relation with it. *I do the relating*. I posit an objective combination as willed change just as I posit a sense-content as a willed image. What I know I must assert; it must come from me as *act*. Only in this way does the plurality become apprehended. This is the full meaning of original synthetic unity of apperception. The core subject is act, act can own, but it can only own the results of act. Acting can make an object that it knows perfectly—*itself*, otherized as a

creation. This is how an intuition acquires the unity required for being an object of a unitary subject—the plurality must flow from one act.

It is by positing imaginary contents *and* by positing a way-of-change that the pluralizing form of reality-reception finally becomes the property of a unitary subject. Act must be present at every point-moment that I know because *awareness* is everywhere and every-when that I know, and act is prior to awareness since the subject can apperceive only through asserting “this is P,” where *P* is the rule for making an emulation of *this*. Intuition must be vulnerable to act in order for awareness to be possible—awareness is image-making. But awareness of change is no exception to the criterion of image-making, and so the subject’s act must create not only imaginary *contents*, but also the “image” of their *change*. To make change meaningful, I have to be able to emulate it myself.

The result is a synthesis of intuition that is consciously performed, i.e., one that illuminates the intuition’s spatial and temporal atomization by generating it internally, thereby generating an objective plurality-in-unity that I thoroughly know and apperceive with awareness of myself as subject. This is called a **determinate** or **formal intuition**. A determinate intuition is one that has been synthesized with full consciousness, i.e., under will. It is a product of blind synthesis that I have *re-constructed* consciously, i.e., by following an understandable procedure. It is, Kant says, “possible only through the consciousness of the manifold’s determination by the transcendental act of imagination (i.e., by the synthetic influence of understanding on inner sense)—the act that I have called figurative synthesis” [B154].

There is a difference between an encountering *simpliciter* and an encountering that includes the consciousness of myself as subject of awareness. I can encounter a line, a circle, a room, and even time as given objects without consciousness. These have been

pre-synthesized by imagination under the spontaneous power of understanding, but automatically and unconsciously. But I can also encounter them as *understood* objects. Doing this requires re-creating them under conscious direction.

Drawing, describing, placing

Kant now gives us examples in which this activity of knowing-by-making must be carried out consciously. We commonly do this, and Kant gives us examples that are familiar to us: “We cannot think a line without *drawing* it in thought. We cannot think a circle without *describing* it. We cannot at all present the three dimensions of space without *placing* three lines perpendicularly to one another from the same point” [B154]. This is how we are to understand synthesis that is *consciously directed*. Remember that the transcendental synthesis that produces the generic physical object is *blind*: “Synthesis as such, as we shall see hereafter, is the mere effect produced by the imagination, which is a blind but indispensable function of the soul without which we would have no cognition whatsoever, but of which we are conscious only very rarely” [A78/B103]. But it *must* be possible to carry this out consciously, because it must be possible for the *I think* to be actually thought.

All combination is an act of understanding whose form is the structure of judgment. Since drawing a line is the elementary act that synthesizes the realm of intuition, and since all *understood* combination is determined by the functions of unity in judgment, line-drawing involves “This (S) is P.” Kant goes on: “And even time we cannot present except inasmuch as, in *drawing* a straight line (meant to be the externally figurative presentation of time), we **attend** merely to the act of the manifold’s synthesis whereby **we successively determine** inner sense, and thereby attend to the succession of this determination in inner sense” [B154, boldfacing mine, here and below].

This is an important passage. It shows that the act of line-drawing is not mono-semantic—it can produce more than one kind of sense. Here, the act of line-drawing receives a determination that is independent of and prior to the act itself. This is my *intentionality*. When I make space, I attend to one aspect of line-drawing—its *extendedness*. Much else is going on when I draw, but this is not what I am attending to *when I set out to draw a space*. I *intend* to make space, prior to my act. This is what Kant calls synthesis *under a rule*. The rule is a premeditated plan. I make a dimension of extension by drawing a line and attending to what can only be the *genesis* of the attribute of extension. Conscious synthesis is self-change, and the kind of self-change I make determines the sense of the product of synthesis. Here, the change that I attend to is the kind that yields difference in position. The change I produce is that of *cumulatively adding* the spatial way-of-separation into unified spatial extension.

But there are other possible objects of my attention when I draw a line. Kant gives one in the passage above: “the act of the manifold’s synthesis whereby **we successively determine** inner sense, and thereby attend to the succession of this determination in inner sense.” When I draw a line, something else is going on. My contents (called “inner” contents) are passing away. But I *can* know this only because automatic synthesis has pre-combined these passing contents into a unity that I can then traverse under conscious direction, thereby first producing time itself.

By “passing away” I mean the a priori dimension of difference presented by sensibility that *becomes* succession but cannot be called that yet because sensibility presents no unity. I can add passed moments together only if I can produce the gap between them. So I create “passing away” myself—I intend a change in myself called *next*, or *succession*. “This is next, now this is next, ...” But next cannot be presented as a *unity with* the past unless the passed moments are made present. This is Kant’s primary

synthesis, the one given special attention in the A-Deduction, called **transcendental synthesis of reproduction**.⁵⁴ I reproduce passed moments by producing points as I draw a line—points *that are retained in the same space* as the point I am positing *now*. Drawing adds new points to the old, which are retained in the form of *spatial* difference. Every *next* moment becomes an infinitesimal distance *forward*, and every *duration* some finite *distance*. But for this to be a unity, this relation of infinitesimal distance must be produced by *one act*. This is the act of moving a geometrical point.

In short: I construct time by attending to how I might *produce* “passing away” for myself, in such a way that the passing is presented by being retained and combined with the present.

Spatial distance presents time, thanks to the mediating act of moving an identical point. Moving is both unity and plurality—the mover is a unity that adds together the occurrence of passings-away *consciously* by producing spatial difference. Time receives its sense from motion, which is a way of attending to the act of line-drawing: “what first produces the concept of succession is **motion**, taken as act of the subject (rather than as a determination of an object)* and consequently as **the act** whereby we determine *inner sense* according to its form” [B154–55]. By motion Kant means specifically the act of moving an imaginary point: “**this intuition is that of the motion of a point in space**; solely the point’s existence in different locations (as a succession of opposite determinations) is what first makes change intuitive” [B292]. Thus *moving a point* is the fundamental act of *our* figurative synthesis. It is the needful hybrid act that is both one and many, both act and intuition.

⁵⁴ Despite its name, the transcendental synthesis of reproduction belongs to the **productive imagination**. The productive imagination produces the first unities in intuition, the generic physical objects, that allow universals to be abstracted and the *associative* concept-connections of the *reproductive* imagination to take hold.

Kant reminds us in the footnote at B155 (indicated by the asterisk above) that the motion here is one that is purely imaginary, that is, self-produced through conscious activity, not an encounter with physical motion, something that can only emerge after the process of synthesis is complete. Kant is here talking about motion as self-made, as the very *process* of synthesis itself. Moving a point through spatial difference extends my identity, the identity of my unitary act of line-drawing, through space, and so unifies it objectively. It unity comes from the fact that the act precedes the plurality. Unity can only come from the subject. This is done by having the subject produce spatial difference by inducing change internally. Apprehension is *prehensive*: it anticipates the plurality that it intends to collate by producing it internally as change. It anticipates the next point by generating it—from unity. So it is misleading to say that a given plurality is gathered after the fact of their arrival. Rather, I have already generated this plurality from a unity. I have generated the empty framework that is ready to receive them by spinning it out of my identical activity.

We should note that the identity of the moving point, while only mentioned by Kant in the context of time-making, is also the basis of the *unity* of space, and the conception of magnitude as a unified totality:

Motion of an *object* in space does not belong in a pure science, and consequently not in geometry. For the fact that something is movable cannot be cognized a priori, but can be cognized only through experience. But **motion taken as the describing of a space is a pure act of the successive synthesis**, by productive imagination, of the manifold in outer intuition as such, and belongs not only to geometry but even to transcendental philosophy. [B155 fn. 283]

The intended identity of a moving point is what gathers together adjacent points into, not a pair of points, but a *span*. Thus the notion of an identity-through-time is the essential ingredient in the concept of space as a continuity rather than as a set-unity.

I consciously produce time as a line when I draw by moving an imaginary point while attending to the fact that my act of motion is the active analog of the passive succession of inner sense. Because I am the unitary agent that produces the change-in-position, I own every point along the line as I pass through it in imagination. The moving point combines the passing contents of inner sense into slices of a unitary time by *producing* the fact of passing itself, as a motion that, because it is a single act, is the source of contents both passed to the present, in the unifying presentation of a line: “by no means does the understanding already find in inner sense such a combination of the manifold; rather, the understanding *produces it*, inasmuch as the understanding *affects* that sense” [B155]. To “affect” inner sense *at all*, I must *posit* a content (elementally, an imaginary point). To affect the *combination* of inner sense, I must posit the active analog of passing-away by producing change-in-position. This act brings unity to what would otherwise be a plurality of discrete moments. The unity comes from the fact that I am positing *only* a change in position, the point that moves is posited by me as being identical across spaced and through time. The act of moving a point that is continually identical is what originally produces the category of substance.

The act of moving a point is in fact the interface between the two sources of selfhood—spontaneity and receptivity. It is this act that explains how “the *I* who thinks [can] be distinct from the *I* that intuits itself, and yet be the same as it by being the same subject” [B155]. The active and the passive are bridged by introducing two mediating elements—the rule of combination (on the side of agency) and the way-of-separation that must be combined (on the side of receptivity). The act of understanding employs invariant rules, and reception by intuition contains a priori a certain way-of-separation, which is space. The identity of the agent is realized by the fact that I intend the motion of an *identical* point. This bestows the resulting imaginary intuition with unity. Identity of

act unifies space as an identical point that merely changes position, rather than its identity. This change is forced by the I that thinks, while the way of change is presented as intuition (as position). *Line-drawing generates time as space.*

This conscious construction of time by means of the rule “move a point while *attending to the succession of this determination in inner sense*” is an example of figurative synthesis. This rule is also, as we will see in the Schematism, the schema of **magnitude**: The schema of magnitude “contains and is responsible for the presentation of ... the production (synthesis) of time itself in the successive apprehension of an object” [A145/B184].

The special section on line-drawing ends by tying all this to the problem of self-knowledge, i.e., my ability to be an object to myself. On one hand I am active agent—I *think*. To think for Kant means to advance a rule that can create an image that emulates an intuition. On the other hand, my contents are sensory states in inner sense. When I cognize my body, I am actually tracing over appearances in outer sense, and “space is already accepted as being merely a pure form of the appearances of outer senses” [B156]. What about the cognition of my temporal existence? “For as regards **time**, which after all is not an object of outer intuition at all, we cannot present it to ourselves except under the image of a line insofar as we draw that line; without exhibiting time in this way, we could not cognize the singleness of its dimension” [B156]. Line-drawing generates time as space. Time is intelligible only as a line, because the very meaning of time does not exist in thought (as rule of emulation) unless I know how to consciously produce the object (time, in this case) myself by following a procedure. What is produced is not just an analogy, but time itself.

Passing is given change; movement is consciously directed change that correlates passing with position. By means of line-drawing, time becomes presented as a unity and

as a structure for the first time. Time is a unity; passing is not. I can only add passed moments together by drawing a line.

A moment's reflection reveals that all the attributes of time are derived from space. The one-dimensionality of time, or example, is unquestionably one of its essential characteristics. Yet "one-dimensional" is a spatial presentation. In fact, time borrows *all* its formal attributes from the form of space:

Likewise, in seeking for all inner perceptions the determination of length of time, or again of time positions, we must always get this determination from what changeable features are exhibited to us by outer things. Consequently the **determinations of inner sense must be arranged by us** as appearances in time in precisely the same way as the determinations of the outer senses are arranged by us **in space**. [B156]

Kant ends his special section on line-drawing by tying all this to the self's power of self-cognition. Cognition is ostensive judgment—judgment that aims toward the passing plurality of pixel arrays. Treating the ultimate container of this plurality, Kant says:

I fail to see how one can find so many difficulties in the view that inner sense is affected by ourselves—of which every act of *attention* can provide us with an example. In such acts the understanding always determines inner sense, in accordance with the combination that the understanding thinks, turning it into the inner intuition that corresponds to the manifold in the understanding's synthesis. Everyone will be able to perceive in himself how much the mind is commonly affected by this. [B156 fn. 292]

Kant here identifies the act that appropriates sensibility via imagination with the common act of *paying attention*. Merely noticing this rather than that affects the contents of my inner sense. By noticing one thing rather than another, I bring it to the unity of apperception and thus of judgment. It is *attention* that determines the matter and product of synthesis when synthesis is conscious.

§25—[UNTITLED]

The self's activity of synthesis as it is carried out by understanding, as the activity of combining *any* kind of presentation, yields not cognition, which requires sensible intuition, but mere thought: "I am not conscious of myself as I appear to myself, nor as I am in myself, but am conscious only that I am" [B157]. On the other hand, cognition of ourselves requires not only the generic, all-consuming synthesis of understanding, "but requires in addition a definite kind of intuition whereby this manifold is given"—that is, the way-of-separation through which sensation is originally received as a plurality.

Self-cognition is necessarily cognition of self-as-appearance, and the categories do not apply to the active, core subject-as-*appetitio*. The objective self is an object made of inner states. The core self, however, is never an object, but only the "thought" that is the telos of generic synthesis by understanding in isolation. Kant calls this the self-as-*intelligence*. Its only attribute, as we have mentioned, is act: "This intelligence is conscious solely of its power of combination" [B158–59]. All other self-knowledge is knowledge of inner states combined into the flow of my inner reality. The self that I cognize is a temporal stream of states.

All this is explained succinctly in the footnote at B157:

The *I think* expresses the act of determining my existence. Hence the existence [of myself] is already given through this *I think*; but there is not yet given through it the way in which I am to determine that existence, i.e., posit the manifold belonging to it. In order for that manifold to be given, self-intuition is required; and at the basis of this self-intuition lies a form given a priori, viz., time, which is sensible and belongs to the ability to receive the determinable. Now unless I have in addition a different self-intuition that gives, prior to the act of *determination*, the *determinative* in me (only of its spontaneity am I in fact conscious) just as *time* so gives the determinable, then I cannot determine my existence as that of a self-active being; instead I present only the spontaneity of my thought, i.e., of the [act of] determination/ and my existence remains determinable always only sensibly, i.e., as the existence of an appearance. But it is on account of this spontaneity that I call myself an *intelligence*. [B157 fn. 296]

§26—TRANSCENDENTAL DEDUCTION OF THE UNIVERSALLY POSSIBLE USE IN EXPERIENCE OF THE PURE CONCEPTS OF UNDERSTANDING

Kant now lists the main stages of his argument establishing the a priori origin of the categories. The **first** was the argument of the Metaphysical Deduction, where “we established the a priori origin of the categories as such through their complete concurrence with the universal logical functions of thought” [B159]. The **second** was the argument of the Transcendental Deduction, where “we exhibited the possibility of them as a priori cognitions of objects of an intuition as such (§§ 20, 21).” The **third** stage is Kant’s current topic:

We must now explain how it is possible, through *categories*, to cognize a priori whatever objects *our senses may encounter*—to so cognize them as regards not the form of their intuition, but the laws of their combination—and hence, as it were, to prescribe laws to nature, and even to make nature possible. [B159]

This list of stages is helpful—it shows that Kant is moving in a clear direction. First, it shows that his theorizing moves *outwards* (from the subject to the object). Second, it shows that his theorizing moves from *generality to specificity*—from a combining of *all possible* intuition into a combining that is, as we shall see, specifically spatial.

Categories are based on rules of thinking—the Metaphysical Deduction

The first movement is one from the subject of knowledge to its object. We begin with the subject, in accordance with Descartes’ order of discovery: what is known best is what is closest to the knower. What can be known a priori must lie in the knower. When I know things that *must always* hold for the object *and know that I know this*, my conviction of this necessity, Kant says, indicates that it is flowing from some condition of my ability to be a subject of knowledge. The conditions for knowing *any* object are conditions that hold necessarily and universally of *all* objects.

Categories are realized as objective characteristics—the Transcendental Deduction

The Transcendental Deduction takes the next step outwards towards sensible intuition “as such.” This is an important step because it deals with the highly problematic interface of spontaneity (understanding) and passivity (intuition).

What makes the objective reality of the pure concepts possible is the fact that it is actual and necessary. It is actual because we do, in fact, have apperception of intuition. So it must have been synthesized by the understanding—by definition of apperception. Synthesis by the understanding is also a necessary condition—I could not have apperception of intuition otherwise. All Kant has to do is *explain* it. But this has still not yet been done by § 21 (which is his reference in this section). All that is explained is what *must* happen—that is, Kant has only refined the problem. What must happen (and what still requires explanation) is that “everything manifold, insofar as it is given in one empirical intuition, is *determined* in regard to one of the logical functions of judging, inasmuch as through this function it is brought to one consciousness as such” [B143]. Unification occurs *via* judgment. Unification has the structure of judgment as its final cause—it must end in the *thought* “S is P.” But unification is cognition only when the content of judgment is an intuition, so unification must end in a slightly different form: “This (S) is P.”⁵⁵ The result of synthesizing intuition into conformity with “This (S) is P” is the objective presentation or “realization” (the term used in the Schematism) of the components of judgment: *S* and *P* combined by the copula. Understanding’s form has

⁵⁵ Remember that the unity in judgment serves the higher unity of reason—the unity of syllogistic inference. Reason requires a universal in the subject-position of judgment in order for inferences to be possible. But when judgment is ostensive, or points at intuition, the subject-position refers directly to intuition—to a substantial body. The kind of this object need not be placed in the subject-position, and in fact is originally generated out of the predicate-position. When I learn the rule of, say, *chair*, I do so by comparing *this* with *that*, until I realize that “This is a chair.”

become manifest in intuition by “determining” it. This determination produces a generic object whose aspects are the original instances from which the categories are abstracted.

How the categories are realized in the generic physical object

Now, in § 26, Kant begins the third and final stage of his explanation, that of explaining how laws of cognition can determine not only “the form of their intuition, but the laws of their combination—and hence, as it were, to prescribe laws to nature, and even to make nature possible” [B159]. Any object of our human apperception is a unified span across space and time. But there is more to physics than this. Objects are lawful in their time-evolution. The possibility of mathematically lawful mechanics must now be explained. This is done by the threefold synthesis.

Apprehension accomplishes the spanning just mentioned. I gather points across the simultaneous plurality of outer sense and make a unitary space, and I also gather moments across the passing of this given plurality and make time. Prior to these syntheses of apprehension, I can think neither space nor time. There is only the plurality that is given and the plurality of passing. In these pluralities, however, there is something that I understand and can therefore consciously combine, i.e., as an assertion in judgment. The something that I understand and can combine is my a priori familiarity with what Kant calls the “mere *forms* of sensible intuition” [B160]. The forms of intuition are ways-of-separation of which I am not yet apperceptive, but can be. To be apperceptive, I must combine the elements contained in these mere forms by traversing the “gaps” that these forms are made of.

Of course, I am only aware of space and time *post*-synthesis. Space and time are never presented *to me* “merely as *forms* of sensible intuition, but as themselves *intuitions* (containing a manifold), and hence are presented with the determination of the *unity* of

this manifold in them” [B160]. The forms of intuition are conjectures, never actually known, but required by Kant’s doctrine that any understood object is one that has been combined by self-activity. There must be a plurality for me to combine if I am to know it. So space and time must be atomized, merely in order for me to get the benefit of unifying them myself. Every object of understanding is a combination that I have carried out. Every object that I *consciously* understand is one that I have emulated as judgment. Here, this emulation is the conscious act of making space (or time) by the understanding. Thus “our ability to produce presentations ourselves” is required in order to have something to *assert* (produce) in the act of judgment, which produces apperception, makes me a knowing subject of an intuition, and permits the necessary “I think” across the plurality.⁵⁶ This is the reasoning behind the important footnote at B160:

Space, presented as *object* (as we are actually required to present it in geometry), contains more than mere form of intuition; viz., it contains also *combination*, of the manifold given according to the form of sensibility, into an *intuitive* presentation—so that the *form of intuition* gives us merely a manifold, but *formal intuition* gives us unity of presentation. In the Transcendental Aesthetic I had merely included this unity with sensibility, wanting only to point out that it precedes any concept. But in fact this unity presupposes a synthesis; this synthesis does not belong to the sense, but through it do all concepts of space and time first become possible. For through this unity (inasmuch as understanding determines sensibility) space or time are first *given* as intuitions, and hence the unity of this a priori intuition belongs to space and time, and not to the concept of understanding (see § 24). [B160 fn. 305]

Space and time *themselves*, the very containers of any possible sensible reality, are not objects of my awareness until they have been combined in the synthesis of

⁵⁶ Apperception is understanding, which is act. At the opening of the Transcendental Logic, Kant defines understanding as (1) “our ability to produce presentations ourselves, i.e., our *spontaneity* of cognition,” (2) as “our ability to cognize an object through [given] presentations (and is the spontaneity of concepts),” and (3) as the spontaneity through which “an object is *thought* in relation to that [given] presentation (which [otherwise] is a mere determination of the mind)” [A50/B74]. In the Metaphysical Deduction, this is taken in the broadest sense, which is how Kant makes good on his claim that understanding is independent, and works according to its own agenda, which is intellectual synthesis, an acting-out of “S is P” in the service of apperception.

apprehension. This is surely an economical way to establish the objective reality of the categories: since all objects appear in space and time, making synthesis (and the categories) necessary for awareness of space and time themselves also makes the categories necessary for all objects, which after all are constructions out of appearances that originate as empirically filled point-moments.

So the space and time that Kant has been referring to all along, it turns out, have not actually been forms of intuition, but constructions. We only know space and time post-synthesis. This puts the Transcendental Aesthetic in a predicament, since its very purpose was to treat the mere forms of intuition—in isolation from understanding. But we cannot even *talk* about space and time apart from understanding. The forms of intuition “precede” the concepts of space and time as the matter *for* synthesis, a matter that we can never access except *through* synthesis. The forms of intuition are nothing but ways-of-separation shared by the elements belonging to some mode of sensibility—inner or outer. But space and time are nothing for me until synthesis has occurred. Of course, they *are* unities for me, so synthesis *has* occurred, “and since experience is cognition through connected perceptions, the categories are conditions of the possibility of experience and hence hold a priori also for all objects of experience” [B161].

How combination of space and time is determined by a function of unity in judgment

Space and time are combinations for apperception accomplished through the effort to understand, which is actualized by asserting a judgment, and so governed by the judgment-forms. The *I think* is possible only where combination yields something that can be asserted, because epistemic consciousness arises only as the awareness that some object “is P.” I apperceive by asserting a kind, by producing a unity-over-plurality, in the

structure of judgment. This means that the judgment-forms (the pure concepts) are at work. Space and time themselves, Kant has just said, are no exceptions.

Kant then gives some examples of how pure concepts play a role in figurative synthesis. Take the (visual) cognition of a house. Intuition provides me with some cluster of point-data—points of color that differ at a boundary from points of another color. I combine this cluster in the synthesis of apprehension. I move across the point-data by line-drawing, by moving an identical point across the way-of-separation that makes the manifold (cluster) of points *a manifold*. In this way, I consciously “move” my identical act of affecting my own passivity and emulate the figure of the house-to-be by tracing over it in the imagination and “determining” it myself: “I draw, as it were, the house’s shape in conformity with this synthetic unity of the manifold in space” [B162]. I perceive a house by making one.

Magnitude: the rule of space

But all emulation of understandable objects is emulation that has as its rule some function of unity in judgment. Intuition provides an a priori way-of-separation, and understanding provides a pure concept that combines it. The objectified pure concept here is **magnitude**: “But this same unity, if I abstract from the form of space, resides in the understanding, and is the category of the synthesis of the homogeneous in an intuition as such, i.e., the category of *magnitude*. Hence the synthesis of apprehension, i.e., perception, must conform throughout to that category” [B162]. This shows that the category of magnitude is required for the presentation of any *figure*, and therefore any *body*.

The fact that the synthesis of apprehension is really the synthesis of magnitude is something that the A-Deduction never revealed. The A-Deduction dealt in depth with the

synthesis of apprehension, but its mode of exposition was so firmly tied to the narrative of British Empiricism that the identity of apprehension and magnitude was never explicit. In the A-Deduction, Kant was focused on showing how the empirical theory of concept acquisition actually depended on a priori processes that the Empiricists neither explained nor acknowledged. Even the mere apprehending of empirical point data, Kant argued, depends on a prior apprehension of the fields that contain these pluralities and bring them to a unitary subject of knowledge. Now we find out that this same act of apprehending is actually guided by a pure concept—*magnitude*. And because this pure concept is originally a necessary judgment-form, *magnitude* applies necessarily and universally to all possible sensible objects.⁵⁷

Cause: the rule of time

Kant then gives another example. Take the freezing of water. This cognition of water freezing also involves apprehending point-data into a body, of course. But something else is cognized—a change of state: “I apprehend two states (fluidity and solidity) as states that stand to each other in a relation of time” [B162]. All intuitions are passing contents of inner sense, whose form is time. But the *form* of time, being only the innate way-of-separation provided by outer sense, is nothing to me. Only *formal* time, which has been generated by line-drawing, is an object of apperception. Apperception of the sequence of states in time involves synthesis. The objectified pure concept here is **cause**, and “through this category, when I apply it to my sensibility, *everything that happens is, in terms of its relation, determined by me in time as such*” [B163].

⁵⁷ The rule of synthesis here is *logical* quantification. We will see how magnitude is really the objectification of a “function of unity in judgment” in the next chapter, dealing with Schematism and its associated principles.

In the example, first there is fluidity and then solidity. In the cognition of water freezing, this order is necessary. Unlike pure time, what is presented is not the mere succession of empty moments that contain *possible* data, but lawful succession of *particular* data, which are states, or contents. These contents occur in a determinate order in time—that is to say, they occur lawfully. But for contents to be lawfully ordered in time just means that the position of every content is determined as a function of time. Since this pure concept is originally a necessary judgment-form, *cause* applies necessarily and universally to all possible sensible objects.⁵⁸

Returning to the Herz problem, and announcing the solution

“Now this question arises: Since the categories are not derived from nature and do not conform to it as their model (for then they would be merely empirical), how are we to comprehend the fact that nature must conform to the categories, i.e., how can the categories determine a priori the combination of nature’s manifold without gleaning that combination from nature?” [B163].

Apperception (in understanding, of intuition as such) has judgment as its form. Our outer sense has space as its form, and our inner sense has passing has time as its form. No intuition could come to apperception unless the synthesis of understanding occurs. But we do apperceive intuitions, and that means that they have been synthesized into conformity with the structure of judgment. But this means that intuition has been molded into a unity that can be disassembled and then reassembled as “This (S) is P.” Now, intuition provides *no* combination, only plurality, conditioned by innate ways-of-pluralization that are also ways-of-separation.

⁵⁸ The “function of unity,” or pure concepts, here is the copula. (We will see how the synthesis of cause flows from a “function of unity in judgment” in the next chapter. I will obviously have to explain why this flows from the unifying in the copula rather than the unifying of the “if ... then ...” form of hypothetical judgment.

Understanding, however, is *act*—I say “S is P,” which means I *make it*, namely, as an assertion. I assert my judgment as being *true* of some object, i.e., true by emulation. Taken in isolation, understanding produces emulation by means of intellectual synthesis, something that is prior to and independent of all objective intuition. So the answer the Kant’s question of how it is that “nature must conform to the categories” is again one that tries to show that conditioning by understanding is “not any stranger than how it is that appearances themselves must agree with the form of a priori sensible intuition” [B164]. Both, Kant thinks, are on a par, because both are aspects of the subject’s power of cognition.

Under representationalism, every bit of knowledge is *of* the subject. Objects are made of sensations, and sensations are bits of my self. They are immediate contents; they are *my* contents. Well, given this view, the idea that they arise (in me) in a certain way should not be surprising, and is at least plausible. They arise as points evenly dispersed in a *certain* way. They arise in a plurality that is, to be sure, all *mine*. But making good on the supposition that all data are mine by making them *explicitly* mine can only be done by concentrating on a simple datum. Data are potentially different in content at the level of the spatial point. If I see two colored points side-by-side, I cannot think the object as a unity because I would have to employ a time-taking AND. The subject of *sensation* is eternally in the present moment. This is the *real* referent of the “I think” that Kant says “must be able to accompany.” Remember that Kant has said that even the unity of analytic judgment, which we can reduce to “P is P,” is still an act of synthesis for this very reason: the subject-*P* and predicate-*P* are intended at *different moments*, and are for this reason different “consciousnesses.” [B130 fn. 191].

A truly unitary consciousness can only be ensured by reducing the synoptic scope of my awareness to a magnitude where multiplicity is impossible. This is because if I can

see multiplicity, then I have multiple sense-consciousnesses. If I see something, and think “this part is red, this part is black,” then I do not have unity. I am unitary only when the intuited object is simple.

So being mine is limited to point-awareness. But, in fact, I have available to me other points I can notice as unity. What is it that makes *this* point *different* from some *other* one? This otherness is a way-of-difference, a way-of-separation. Kant says that I can know this way-of-difference a priori only if it is the way that my self *presents* a plurality. A plurality arises, but I know that I (a unity) can be explicitly aware of it all—and I know this because sensibility arises *in me*, and hence I know its way-of-separation a priori. I do this by taking my point-awareness and moving it into an adjacent position—one that is different but *connected* to the current point of my awareness. It is by moving from unity into adjacent unity *under a continuously identical act* that appropriates (apprehends) the a priori way-of-separation into a unity that, yes, is truly mine, as now verified. What for intellectual synthesis is called vaguely “adding” is shown to be, when applied to outer sense, is in fact *traversing while attending* to the fact that I, who am traversing, am continually identical. And “adding” is also shown to be (what is the same thing just said, but from a different perspective) the *production of change* by a rule and a continually identical actor that do *not* change. Plurality is best brought to unity by being *produced* by it. I produce not only combination, but also plurality.

There are two parts to this story, Kant says, and neither of them is really “strange.” The least strange is the idea that my sensibility would have a special feature—i.e., that it delivers a plurality in a certain uniform way, and that this way is invariant and constantly in effect and, so, a necessary condition of all appearances, and thereby a necessary feature of all possible objects. What is *apparently* stranger is that something similar is going on, not with sensing, but with presenting *unities*—such as universals. But

here Kant is not yet talking about this kind of unity. He is talking about magnitude and causality, which are the unities (categories) that I think when I unify empirical point-data qua simultaneous plurality into spatial unity, and when I unify empirical point-data qua passing into temporal unity.

I *understand* physical objects. What does this mean? It means that I unite their ways-of-separation and grasp *them* instead of nothing, or carry out a completely atomized non-grasping of passing pixels. What I understand are the ways in which I a priori imagine the object. Pixels pass, but instead I think a persisting substance. Why? Substance, Kant says, is a way that I combine sensations. What way? Kant is after pure concepts, remember—concepts that *must always* apply, or “apply a priori.” This is as good as saying they apply *necessarily*. But what can be truly necessary about an object except ... the conditions for my being able to know it? The conditions for knowledge are the conditions of truth—knowledge is justified true belief. Truth is identity of object and assertion. To assert truth is to assert in the structure: “S is P.” This is how I understand the object: as an *S* that is *P*. *For this to be possible the object must already be put together in such a way.*

We know how this is done (and how truth, analytic truth, is possible) in the case of intellectual synthesis—the understanding that analyzes-out the *P* and then re-combines it with the *S* in my conscious act of judgment is the *same* understanding that originally brought the object to *my attention* in the first place. And what brought it to attention was my act of constructing a *logical object*. The whole object is the subject, which I make by pushing universals together via logical combination. I extract one and then *say* that it is contained in the compound from which I extracted it.

But here the object is made of sensations. Kant’s “deduction” is really one of *identification*. This is my interpretation of the *argument* of the Transcendental Deduction.

Kant *identifies* the subject and predicate-positions as what rule the ways-of-combination that I carry out in making a physical object, which *is*, in fact, susceptible to “This S is P.” This unified object, under representationalism, can only have been carried out by me. Moreover, since it is consciously understandable, it must have been carried out by understanding, i.e., by the power that understands by asserting “S is P.” This structure contains judgment-forms that allow it to create reasons unified web of syllogistic inference. These also apply to physical objects. And so, as Kant says in the Metaphysical Deduction, “The same function that gives unity to the various presentations *in a judgment* also gives unity to the mere synthesis of various presentations *in an intuition*” [A79/B104]. Thus his thesis that “the laws of appearances in nature must agree with the understanding and its a priori form” is not stranger than the thesis that “appearances themselves must agree with the form of a priori sensible intuition” [B164].

When I combine space by drawing a line I am carrying out consciously something that I must have already carried out unconsciously. Doing this consciously must be possible, because it must be possible for the *I think* to explicitly accompany every point-position that it *can* accompany, which is all those that are mine, meaning available for drawing-through in just this way. To make the *I think actually* accompany every presentation, I simply intend “I think” at every point, not as a *set* of discrete *I thinks*, but as *one I think*—in one *continuous* and *perduring* act of intending. This smears my unity out over space, and so unifies space into an object that is a plurality for one subject. My contents are always passing, however, so the purely sensible *I think* is not only a point, but also a moment. So the same act unifies these point-moments in time as well.

The ways-of-combination that Kant identifies as judgment-forms (*magnitude* and *substance* in the subject-position, *property* and *quality* in the predicate-position, and *causality* as the determination of property-value as a function of time, thereby forcing

appearances into a deterministic or causally necessitated *time-order*) are in fact ways of interpreting the act of *line-drawing*. Line-drawing can be carried out *with different kinds of intent*, and the judgment-forms are precisely, when applied to passing pixels, nothing other than ways of intending the act of line-drawing:

- “We present time sequence by a line progressing *ad infinitum*, a line in which the manifold constitutes a series of only one dimension. And **from the properties of that line we infer all the properties of time**, except for the one difference that the parts of the line are simultaneous whereas the parts of time are always sequential. This fact, moreover, that all relations of time can be expressed by means of outer intuition, shows that the presentation of time is itself intuition” [A33/B49–50, boldfacing mine, here and below].
- “And even time we cannot present except inasmuch as, in *drawing* a straight line (meant to be the externally figurative presentation of time), we **attend** merely to the act of the manifold’s synthesis whereby **we successively determine** inner sense, and thereby attend to the succession of this determination in inner sense” [B154,].
- “For as regards **time**, which after all is not an object of outer intuition at all, we cannot present it to ourselves except under the image of a line insofar as we draw that line” [B156].

But the strongest phrasing of the identity of the categories with interpreted acts of line-drawing is in the Schematism:

- The schema of magnitude “contains and is responsible for the presentation of ... the production (synthesis) of time itself in the successive apprehension of an object” [A145/B184].

The Schematism is where Kant does just this job of describing how a judgment-form serves a rule that, when we follow it consciously, acts precisely as a rule for

interpreting the act of line-drawing. The word he used at B154, in the special section on line-drawing, was “attend.” While drawing, I can attend to a variety of things, different goings-on that occur while the continually identical act that is the referent of the *I think* carries out its work. To some external observer, the act of drawing merely produces a line. But for the agent that draws it, this same act produces the meaning of *time* when the agent “attends” to the successiveness of its act, i.e., the fact that it is now producing an acted-out version of the passing of inner sense. This, I contend, is what it means to “follow a rule of synthesis.”

But the schema that produces time, Kant says, is that of *magnitude*. The schema of magnitude is a special use of logical quantification, the rule that in the domain of universals (and expressed in *non*-ostensive judgment) carries out the whole/part distribution of the predicate over the subject, and is expressed by the operators “all” and “some.” It is the relation of whole/part thought through logical quantification of predicate over subject that is the “rule” that guides the imaginary production of time as the image of magnitude. This “logical” operation is what determines the *way* of reproduction, i.e., *how* I interpret the combinatory act of line-drawing so as to produce a magnitude: I think “this” one line into “some” parts which I then recombine into a countable “all.” This is how the sensible synthesis of imagination is determined by the intellectual synthesis of understanding, and the way in which this intellectual synthesis (“all” and “some” in this instance) is translated for employment towards the domain, not of universals, but of inner sense and its passing pixels:

Now what connects the manifold of sensible intuition is imagination; and imagination depends on understanding as regards the unity of its intellectual synthesis, and on sensibility as regards the manifoldness of apprehension. Now all possible perception depends on this synthesis of apprehension; but it itself, this empirical synthesis, depends on transcendental synthesis and hence on the categories. Therefore all possible perceptions, and hence also everything whatever

that can reach empirical consciousness, i.e., all appearances of nature, must in regard to their combination be subject to the categories. [B164–65]

Every way-of-plurality of which I am self-consciously aware is one that has been unconsciously combined in spontaneous synthesis but which I must also be able to combine consciously, or schematize, in the actual act of judging, by emulating it explicitly. I do this by following the same rules employed in the spontaneous act of synthesis, but now I do so in a conscious act of figurative synthesis, which is just the act of line-drawing performed under the attention-directing influence of a judgment-form. This is the source of my explicit understanding of physical lawfulness. The original “law” is the law that the object be susceptible to “This S is P,” because this structure is what lets the knower arise as an apperceiving subject that is aware of being a subject. Conscious self-awareness (an actually intended *I think*) is nothing other than judgment (which is consciously asserted).

§27—RESULT OF THIS DEDUCTION OF THE CONCEPTS OF UNDERSTANDING

Kant reiterates his Copernican hypothesis and shows why we now know that it is not just a hypothesis: with respect to apperception, understanding has top priority. Anything that is not understood *has no subject*, and is therefore never presented.

Kant has illustrated the objectification of *magnitude* and *cause*. But these illustrations are not yet explanations. That comes in the next chapter of the *Analytic*, which deals specifically with the act of *fully conscious* synthesis, or **schematism**, which is necessary for the final act of knowledge—that of justifying the correspondence (which Kant will call “homogeneity”) between the consciously emulated and the automatically synthesized object, which he calls the **power of judgment**: “as to how the categories make experience possible, and as to what principles of the possibility of experience they

provide us with when applied to appearances, more information will be given in the following chapter on the transcendental use of our power of judgment” [B167].

Finally, Kant argues against an alternative to the Copernican hypothesis. The alternative he offers is that of *pre-established harmony*. Why not say that my way of combining corresponds to the actual (noumenal) way in which an object is in-itself combined? The answer is that this would undermine the justification of necessary truth. Connections for me are necessary only if necessary for apperception. It is necessity-for-apperception which is transferred to the object in order to ensure knowledge of necessity. Only if the presenting *makes* the object can I ensure that what I emulate (and have previously combined automatically) has objective reality. Otherwise, I would know only what is necessary for my presenting, but not also for the phenomenal object. The object must arise *after* my presenting; or, my presenting must be a condition of the object. And showing this was precisely the goal of the Transcendental Deduction.

Chapter 5: the transcendental schemata and their principles

THE ANALYTIC OF PRINCIPLES

The power of rules

“If understanding as such is explicated as our power of rules, then the power of judgment is the ability to *subsume* under rules, i.e., to distinguish whether something does or does not fall under a given rule (is or is not a *casus datae legis*)” [A132/B171]. The understanding is the power of the rules that were discussed in the Analytic of Concepts. These rules were just the conditions for the possibility of asserting truth as “S is P.” They are at work both in the subject’s understanding, as rules of judgment, and in the object of intuition, as rules of physical being. The former are discursive rules of intellectual synthesis; the latter, the spatiotemporal rules of figurative synthesis. Without the former, there can be no epistemic awareness, or understanding; without the latter, there can be no object of knowledge. The rules as they rest in understanding are the judgment-forms necessary for asserting “S is P.” The rules as they are objectified in intuition are the ways in which I must imagine point-moments as being combined in order to *cognize* an object corresponding to “This S is P.” Because these ways-of-combination are necessary conditions for my awareness of the object, the universals that I abstract from them have necessary application.

Kant establishes a priori universals, or categories, by approaching the nature of the object from the side of the subject. The only objects that can exist (for me) are the ones that I can be aware of. If there are conditions of objective awareness, these are also conditions of the objective existence. This is how Kant implements the study of “being qua being,” the principle subject of Aristotle’s *Metaphysics*.

Most universals are kinds that may or may not be instantiated in an object. The categories, on the other hand, *must always* be instantiated. This is because the categories are abstracted from objectified rules that are necessary for bringing any sensible plurality to a unitary awareness. I must be aware of any object as an *S* that is *P*, but I must do so by being aware across various ways-of-separation, i.e., the ones that are being combined so that I can cognize an object of knowledge.

A note on the categories' innateness

We should point out that Kant's language here is not distinct. He uses *category*, *pure concept*, and *rule of transcendental synthesis* interchangeably. This is unfortunate because it obscures the difference between what is original and what is acquired. Recall that, in his response to Eberhard, Kant claims that, although the categories are epistemically a priori, they are not *innate*.⁵⁹

The relation of category and innate judgment-form parallels that of *formal* intuition and *form* of intuition. The forms of intuition are ways-of-pluralization that are innate, part of my power of sensibility. But, as we have seen, ways of sensible separation are nothing for a unitary subject, nothing having objective sense, until they have been combined.

Similarly the judgment-forms are innate rules of thinking, innate rules of apperception, whose form is "S is P." I am aware as a subject when I assert "S is P"; when I am not asserting, I am not self-conscious. But these innate rules of combining are nothing for me without intuition. These rules must be realized first, via figurative synthesis, in intuition. Intuition is the source of meaning. Then, through the process of reflection, I abstract these realized rules as semantic universals—i.e., as the *categories*.

⁵⁹ See Chapter 1, "A note on the process of reflection."

The process of reflection is necessary for generating *any* universal, whether empirical, mathematical, or *pure*. As Kant says in the *Jäsche Logic*,

The origin of concepts as to *mere form* [of generality] rests on reflection and abstraction from the difference of things that are designated by a certain presentation. ... Since general logic abstracts from all content of the cognition through concepts or from all matter of thinking, it can ponder the concept only in regard to *its form*, that is, subjectively only; not how, through a characteristic, it determines an object, but only how it can be referred to several objects. Thus it is not for general logic to investigate the *source* of concepts, not how concepts *as* presentations arise, but solely how *given presentations become concepts in thinking*—whatever these concepts may contain, something taken from experience, or something thought out, or something gathered from the nature of the understanding. This *logical* origin of concepts—the origin as to their mere form—consists in reflection, whereby arises a presentation common to several objects (*conceptus communis*) as the form required for the power of judgment. In logic, *merely the difference of reflection* in the concept is considered. [JL § 7]

The pure concepts are the innate rules necessary for apperception, i.e., the innate judgment-forms necessary for asserting “S is P.” These rules are rules of intellectual synthesis and, when judgment aims towards sensibility, rules of figurative synthesis. The categories are original, in that their referents are products of syntheses governed by innate judgment-forms (the conditions of the unity of apperception), but they are acquired in that these innate rules must first be realized and then generated as universals through the process of reflection, which is how universals are made “whatever these concepts may contain.”

What has been accomplished in the Analytic of Concepts

The Analytic of Concepts told us what the rules are and how they function automatically. What they are was discussed in the Metaphysical Deduction: the rules of synthesis are the forms of judgment. What these rules do was discussed in the Transcendental Deduction: they are rules of synthesis, intellectual in nature, and figurative when applied to sensibility. These rules make logical contents in intellectual

synthesis—logical objects that can serve as subjects of judgment from which we can a priori extract predicates that we then recombine in analytic judgment. In figurative synthesis, they serve as rules for combining inner sense, whose form is time, which is originally produced by drawing a line with a certain intention.

Now Kant compares transcendental logic to general logic. General logic abstracts from content, but transcendental logic is about how the judgment-forms *make* content. The *fact* that they make content (the unity of the generic physical object) has been established in the Transcendental Deduction. But Kant has limited his explanation only to (1) why this content-making is necessary, (2) the *general* theory of how content making occurs. Most importantly, the process of synthesis that makes these transcendental contents is spontaneous—it occurs automatically and unconsciously.

Necessity of making contents

What is necessary is, first, being able to say “S is P.” This is the general condition of any awareness, or understanding. Second, there is the fact that the awareness is of a plurality. In our case, there is awareness of passing pixel arrays. When a unitary subject faces intuition, one subject owns all of the plurality that it is aware of. I am aware of the simultaneous pixels and of their passing away. This is what “I” am aware of so that means that “I” am present at every point-moment (that I am aware of). This “I” is an act—an agent. This is Kant’s way of expressing *noumenal self-knowledge*. We cannot apply the pure concepts to the true self, but Kant identifies it anyway as *act*. So act must be present everywhere “I” am.

General theory of content-making

When making logical objects out of learned (abstracted) empirical contents, the “I” simply posits images via logical combination. This makes one logical complex the

object of a unitary subject. This is intellectual synthesis. But in order to combine the passing pixels into a unitary *sensible* object, the “I” must both posit point-moments and also think them as connected across spatial and temporal separation. I do this by drawing lines across and through these ways-of-separation. This is the means of figurative synthesis.

Positing

Why does figurative (sensible) synthesis require line-drawing? In order to combine a sensible plurality into a unity, I must posit imaginary contents. The “I” acts not only by combining point-data across space and time, but also by affecting intuition in order to posit data of its own, and this positing is a necessary condition of combining. I can only mark the point-moment to be combined by altering its content. To *attend* to a point I must also *posit its content*. To pick out a “this” is to refer to *pure* intuition—to a position in space. This position will be filled with some content or another. For me to pick it out is to be aware that I can change this content. I can be aware of a point-moment only if I can change its content in imagination.

Combining

What about combination in figurative synthesis? How do I connect across the plurality of spatial positions? Only by *moving* a point that I invest with my own trans-spatial and trans-temporal identity. I move a point that I declare to be *the same* at both positions. The moving point is thus an imaginary substance (and in fact the schema of the semantic category of substance, as we will see). Only by moving a point that I have declared as self-identical “through” space can I manifest my identity (the identity of identical act) in space and thereby combine spatial points into a unitary space that is an object for me.

Line-drawing is the literal act of conscious synthesis

Line-drawing is what I do when I want to carry out the basic act of transcendental synthesis consciously instead of automatically. It is by moving an identical point that I emulate the basic figurative activity of transcendental synthesis. This act of conscious figurative synthesis is also, as we will see, the basic activity of transcendental schematism.

When Kant first described figurative synthesis in the B-Deduction, it seemed at first that he might simply be trying to *illustrate* the occult process of figurative synthesis by giving familiar examples. In order to think any geometrical object, such as a line, circle, or even three empty dimensions, I must draw lines. [B154] It seemed the line-drawing, here plainly in the service of *mathematical synthesis*, was meant as a mere *analogy* for automatic and therefore necessary figurative synthesis, something that occurs unconsciously and cannot be known directly. The figurative synthesis governed by the judgment-forms must be *like* the synthesis of line-drawing, but not *identical to it*. But then, in the very next sentence, we realize that these reservations about the line-drawing model are mistaken: “And even time we cannot present except inasmuch as, in *drawing* a straight line (meant to be the externally figurative presentation of time), we attend merely to the act of the manifold’s synthesis whereby we successively determine inner sense” [B154]. Time is a special concept—the content of its sense is mere passing, but its form is spatial and its unity derives from movement, in line-drawing. Time is *passing*—and passing is cumulative. This is the origin of magnitude, we find out in the Schematism, and its guiding judgment-form is logical quantification: *this, some, all*. So here is an example of bona fide transcendental synthesis *and* it is being carried out consciously, through a consciously intended and pre-planned act of figurative synthesis, which posits points to claim ownership of the pure contents of intuition (empty positions, filled with

whatever reality *or I as essentially active agent* provide) or *moves an identical point* to establish its ownership of space, and of passing. This is the very act Kant calls **schematism**, and the rule—now stated in intuition-oriented terms—is called the **schema**.

The special status of time, its being *objectively and semantically* entirely a construction of the subject, was actually revealed as far back as the Transcendental Aesthetic:

We present time sequence by a line progressing *ad infinitum*, a line in which the manifold constitutes a series of only one dimension. And from the properties of that line we infer all the properties of time, except for the one difference that the parts of the line are simultaneous whereas the parts of time are always sequential. This fact, moreover, that all relations of time can be expressed by means of outer intuition, shows that the presentation of time is itself intuition. [A33/B49–50] ***

And we were told twice in the B-Deduction that time *must* be *originally* produced by drawing a line, while “attending” to the fact of the agent’s self-realizing *activity* which matches exactly, as a counter-force, the fact of *passing*, which has the nature of succession. When I *present* succession by moving a point to successively different positions, I am aware of being aware of passing: the ontological referent (act) of the “I think” plays a role. This is synthesis *with consciousness*, something we are familiar with any time we judge. Here, I judge the fact of passing: a *this* that continually changes. I am now aware that *this* “is changing,” and

to make even internal changes [in consciousness] thinkable, we must make time, as the form of inner sense, comprehensible figuratively through a line; and we must make internal change comprehensible through the drawing of this line (i.e., through motion), and hence we must make the successive existence of ourselves in different states comprehensible through outer intuition. [B292]

Change can only be presented, actively, by me, as moving a point as an identity across the sensible ways-of-separation (of spatial and temporal plurality). But to get change *into* presentation, it must be presentable, i.e., spatially. What is not spatially

present in intuition is nothing to me. So I posit—I produce an element of change myself. This is how I imitate change in sensibility, which is produced from without. Then, I do not posit “again,” I retain this posit in spite of the internally registered passing of inner sense. This is what *registers* change as *change*: permanence. So *time* originates as *permanence*, and its accumulation of passing makes it *magnitude*. Also, if the object is presented it is non-infinitesimal in space, and is therefore also a spatial magnitude, or *body*. This complex of constructions is what is “thought” under the subject-position in judgment.

These remarks show that line-drawing is not just a familiar and mathematical metaphor for blind synthesis. Time itself is literally generated by accumulating passed states into a magnitude-of-passing, in a way that presents something. What is presented is my own effort to emulate some judgment-form. I intend *this*, and a point is made—an object for my intention. My act of intending, I notice, perdures through the fact of passing. This is change, but my point is identical. The only way to present change *in intuition* is spatially: it must be a spatial presence. This presentation is *movement*, or change-in-space. Continual intending of the same point results in a line: a history of identical act that registers change by positing permanence, in an emulating counter-force. Continual change is overcome by continual positing. And its sense of continuing makes necessary reference to the fact of passing, and so I “move” an identity (it is the same point) to a different *position* (presenting change as difference and *presenting* this difference, i.e., in space).

This is the result of my effort to intend a *this* for “This (S) is P.” I act and make something: an enduring *topic* of my attention. This is the consciousness I have “through” the subject-position, a judgment-form. Kant says that this makes magnitude in the very production of space and time. Time magnitude is *substance*, and spatial magnitude is

body.⁶⁰ (Recall our earlier realization that the “application of the category” that brings “body” rather than “divisible” under the grammatical subject is no application at all, but the *pre*-conceptual automatic synthesis of a space-spanning body carried out by the intentionality of the subject-position. [A94/B129]) These are the products of synthesis that have the “intellectual synthesis” of the subject-position as their rule. This rule is now rendered in figurative synthesis, through line-drawing. Line-drawing is the basic act that will receive an interpretation by a judgment-form, thereby producing a semantic sense such as *magnitude*, *body*, and *substance*. This is my act of schematism, the way I consciously produce the instance from which a category has been lifted through the process of reflection. I do this consciously, and make time as a *meaning*. Kant calls the schemata of the figuratively applied judgment-forms *transcendental time determinations*.

The act of making meaning is the topic of the Schematism chapter. Actually, Kant already included an example of schematism in the B-Deduction when he explained the process that generates (the sense of) time. The Transcendental Deduction was supposed to be about *automatic* (unconscious) synthesis, but Kant could not help resorting to schematism through the act of line-drawing in order to explain the notion of sensible synthesis. Kant, after all, was writing about synthesis in order to make it intelligible. Schematism is synthesis, consciously performed, in an act that makes the meaning of a category in the same way that, with the rule *red*, I make meaning by making instances. In his line-drawing comments in the B-Deduction, Kant is schematizing for us “out loud,” as it were. The Schematism chapter is all about this act of time-making, and the different ways I intend it when I think the unities intended by the judgment-forms.

⁶⁰ “Permanence expresses time as such as the constant correlate of all existence of appearances, or all variation and of all concomitance” [A182–82/B226]. The schema of magnitude “contains and is responsible for the presentation of ... the production (synthesis) of time itself in the successive apprehension of an object” [A145/B184]; “the pure image of the magnitudes of all sense objects as such is time” [A172/B182].

Time: self-change from identical act

I gather time into a meaningful unity by making it myself, by producing the instance, the intuition, that is the content of its meaning, just as a red instance is the semantic content of *red*. The identity of my act forces me to “make” continually, in a continual imaginary positing that produces difference even as it is identical. I move it. By doing so, I make a meaning. I make something consciously in intuition—an instance of something. I am following a schema: I am making an instance with consciousness. This makes, in intuition, what it is that I “think” in a universal, or a rule for making infinitely many instances. I span across a way of difference according to the intentionality of a judgment-form. “I think” the universal consciously, and carry out in actuality what the rule promises in possibility. This is Kant’s story of transcendental instance-making, making that flows from the intent of “This (S) is P,” through the model of line-drawing, because the ways-of-separation that I can combine a priori are my innate ones, which pluralize as simultaneous (proto-spatial) multiplicity and its passing (in time).

This is consistent with his solution to the Herz problem of how spontaneous products of the understanding can refer to extra-mental reality: the object in intuition can only be known a priori, by “pure concepts,” if this object is *also* internalized. If intuition and concept are *ontologically* external to each other (one being subject-produced the other being produced extra-volitionally), then the problem of representationalism is not solved. The intuited object must be internal as well. So Kant makes space and time internal. These are the “a priori manifolds,” under the power of imagination. Reality is received by them, but I can make in them. In fact, I make them themselves.

All this making yields a priori knowledge in the form of magnitude. I unify and divide space and time into numbers, and this is a priori knowledge: truths that would never be false. Necessary truth is where the knowing subject is *certain* that “S is P”

because it makes not only the “thought” that is asserted (the imagination-based meaningfulness of concepts in logical combination), but also the instances that can be asserted. This is truth by virtue of meaning. I make the object when I make the concept, as a potential imaginary instance. In every case, I am spanning time, as my contents pass away. I make an object that counters this pluralization by being thought as a unity: a *this*. From here unity grows through space and time, as I consciously span what understanding has already combined for me, automatically. I think the referent of the subject as *really extended* in space (a body) and *really extended* in time (a substance). Both of these extensions can be meaningful only if I make them, by the definition of meaning as rule of instance-production.

Meaning for Kant is a verb—it is the ability to produce an emulation of a sense content. I see green without “knowing” it. But then I compare different (green) objects. I notice something remarkable—they are different but have something in common. This I then abstract, and generate a “universal.” But the universal “green” is only a name, and conceals the fact that what “green” *means* is really a rule that lets me posit any green image that I could ever receive. Act has taken over reality, or sensation, by covering many instances by means of “thinking” a rule of image-production.

The same holds for “pure” meanings—ones I make myself. *Triangle* is an example of a pure meaning, as is *twelve*. I make these out of my innate forms of pluralization, space and time. And I make them by positing points and lines in the imagination. These are the particulars, or images, that give these words *meaning*, just as green instances carry the meaning of *green*, the rule that makes them.

Each of the pure concepts is a rule of judgment. I think the subject-position *itself* in a certain way. It has, as Kant says, a peculiar “logical meaning.” Now this we are familiar with from general logic. The rules that count are ones that make truth and ones

that alter truth predictably. The rule that makes truth is “S is P.” The rules that determine truth-*value* a priori are *some*, *all*, *is*, and *is not*. Since these are conditions of predication, and thus of kind-awareness, they are also conditions of awareness generally.

“I” extend across space and time through line-drawing. “I” am aware by means of “S is P.” Kant’s theory of transcendental synthesis is simply the outcome of combining these necessities. I must draw lines to span any kind of spatiotemporal way-of-separation. And spanning such separation is exactly what I must do before I can say “This (S) is P” towards intuition. The rules of judgment, then, must be ways of interpreting this act—just as Kant has already said about the production of *time*, which, as we will, see was actually an example of figurative synthesis under a pure concept.

What the Analytic of Principles will now do

Now the Analytic of Principles is about “the ability to *subsume* under rules.” This is the awareness of meaning before the act of application. Meaning is awareness of the rule that makes the instance. The procedure of making is called **schematism**, the rule, the **schema**. The schema of *green* is a rule that knows to make a hue, and knows how to limit this. The pure schema is a rule of judgment, applied to the act of line-drawing.

The pure concepts will eventually mean things. They will become **semantic categories**—the words *substance*, *quality*, *property*, *quantity*, and *causality*. These are a priori applicable—there is *no* object of intuition that can lack them. How is this possible? Because these are rules that make the object in imagination, a sense-emulation, that I need to assert truth towards sensibility, towards *reality*.

Reality has an a priori form: spatial and temporal. Kant drops the spatial because time is a larger container than space (moreover, space is permanent and so space rests in time, as substance). So I face an a priori *form* of plurality—the form of inner sense. This

is what I must combine in order to have a sensible object. Why? Any object must be capable of multiple predication.⁶¹ An object is *F AND G AND H*. It is a plurality in unity that I can resolve and recombine. This is done automatically by the “faculty of rules”—understanding. But now Kant is interested in schematizing, in the conscious act of making meaning. What do I do to consciously make an object of multiple predication that is sensible? I must have a perduring one. Saying “AND” takes time. This is the meaning of the referent of the subject-position. I make it by making time (by line-drawing) while attending to what *remains identical through the fact of passing*. This is me, my act, a posited point, my grasping of a formal point-moment, and I posit it as temporally identical. But where is time? I make change originally, Kant says, through *movement*. [B292] I move the point, and in this way produce an “image” of passing, the meaningful instance that I must make in order to think a meaning. So I make passing actively by moving a point, and I if I attend to the *identity of the point*, I thereby invent from scratch the semantic value of *substance*. This is the process of **transcendental schematism**. This is the process whereby I make meaning internally. This is the meaning of the semantic category. This allows my ability to “*subsume* under rules.”

It is the same process I use when I schematize empirical universals, such as *green*. I make an instance, following a rule. The rule for *green* is “make an image between yellow-green and blue-green.” The rule for *substance* is “draw a line and attend to identity.”

⁶¹ See especially Dieter Henrich, “Identity and Objectivity: An Inquiry into Kant’s Transcendental Deduction,” in *The Unity of Reason*, ed. R. K. Velkley, trans. J. Edwards (Cambridge, MA: Harvard University Press, 1994). Henrich argues that multiple predication is a necessary feature of an ostensive judgment: “a conjunction of various predicates relating to the same subject concept is a possibility necessarily inherent in the form of the elementary statement. ... insofar as these particulars are addressed in the form of the categorical judgment, it is *assumed* that they are endowed with more than one character. ... Not only the combining of subject and predicate but also the very thought of the subject includes the notion of plurality-in-unity” (p. 150). The “cognition which is called ‘experience’ is necessarily oriented toward a conception of phenomena in which many properties are attributed to a single object” (p. 151).

The schematism is the continuation of Kant's line-drawing examples from the B-Deduction. He has already explained the schematism of *magnitude*, he just did not tell us he was doing so. Now he goes through the complete list of judgment-forms. This is not done clearly. From the Metaphysical Deduction we learned that there are two sensible functions of unity contained in the subject-position: substance and quantity. The subject term is both condition of multiple predication and something that I can a priori "think" as a whole or in part. The Metaphysical Deduction also told us that Quality is an operation on the predicate-position. These grammatical assignments are never explicitly made in the Schematism chapter.

The process of schematism is about making meaning ourselves—making possible predicates or kinds. But in the Schematism chapter these *kinds* are necessary conditions of being aware of something that is *P* in intuition and thus of claiming truth and knowledge. They are conditions of the unity of apperception: one "I" aware of many point-moments, leading to one interrelated system of point-data that I can say "This (S) is P" about. The pure concepts are ways of unity that permit empirical kind-awareness, awareness of "is P" in intuition. But knowing that something "is P" means knowing how to produce the instance. The "pure kinds" that are the referents of the categories are objectified functions of unity in judgment—imprints of rules that guide the combination of point-moments automatically by understanding. But now there is something *objective*. I am aware of it as a kind, meaning I can produce a matching instance with a rule. The Schematism chapter is about how I draw a line in order to make these kinds of sensible unity. I do this by drawing while attending to what I "think" in the judgment-form that is the rule for interpreting my act of drawing. Doing this is what makes the imaginary instance of sensible synthesis. The judgment-form is the rule for constructing the unity-

kind that I will eventually abstract as a semantic category by comparing physical objects and noting the *pure* universals that they share.

The five (valid) schemata

I have chosen only five of Kant's group of categories. In fact, it is not even clear how many he has. As Seung has pointed out, he gives three different counts— twelve, eight, and four. (Seung, "Kant's Conception" 118–19)

In the Metaphysical Deduction, Kant lists twelve categories, corresponding to the twelve judgment-forms given in the Table of Judgments. Here, Kant portrays the pure concepts as judgment-forms *analogous to* the forms of general logic. From these forms he immediately derives various semantic concepts of object—the categories. But the details of this logical derivation of the categories are not explained until the Schematism chapter.

In the Schematism, Kant constructs two *new* tables of schemata that are not only numerically incompatible with each other but also with the original Table of Categories developed in the Metaphysical Deduction. While the original table contained twelve unschematized categories, Kant's two new collection of schematized categories contain only eight and four. The first collection (A142–44/B182–184) eliminates the six categories under the Quantity and Quality headings and replaces them with the names of the headings themselves. The second collection (A145/B184–85) eliminates the six categories under the third and fourth headings (Relation and Modality) and again replaces them with the names of the headings. Is there a definitive set of categories? If so, how should we determine it?

I have chosen the valid schemata by inference from the rules of truth-making. The rules of truth-making are the judgment-forms listed in the Metaphysical Deduction. But

only some of these actually are necessary for awareness that brings sensibility into line for the sake of abstracting universals, in truth-functionally manipulable relations, into the web of rational inference which is the logical final cause of knowledge: knowledge of a unified system of objectively real concepts in a priori or analytic relations. This disqualifies *disjunction* and *material implication*. While these are useable in (and in the case of material implication *necessary for*) for the web of reason, they are not necessary for awareness of “This (S) is P,” the means by which I appropriate an image-making rule, or schema, from my sense experience. The pure concepts are the necessary conditions of *kind acquisition*. The pure concepts are the rules of unity necessary for abstracting universals, and for being aware of something that “is P,” and are therefore necessary for *all possible* objects. These are the instances of unity that my understanding creates which allow me to be apperceptively aware as an “I think.” The “I think” always has an object. This is how Kant makes self-awareness, which I must always be able to assert explicitly (so that it “accompanies” my awareness), into a condition from which other necessity flows. I am a unitary subject, aware via “This (S) is P,” and a unitary subject across space and time, *and* a unitary subject across the ways-of-combining point-moments into an object-kind *S* that has the property *P*. When I make this automatic act of understanding intelligible by carrying it out myself, I make the “meaning” of the object that can be an instance of predication. That is to say, I make a physical object. I will show that since all of its essential features are instances of interpreted line-drawing, they are also magnitudes, with the result that the copula is then the relation between two magnitudes. The **subject** thinks a *body* (spatial magnitude) that is also a *substance* (temporal magnitude, and time is the very *image* of magnitude, as we will see). The **predicate** thinks a *property* that changes as an instance of a *quality* whose meaning derives from its position in a *continuum of values*. The **copula** thus relates time-magnitude (a position in

time, such as “now,” that is a number) to state-magnitude (a value on a continuum of quantified quality), and this relation, as a relation between “terms,” is identical to an algebraic relation that determines state-value as a mathematical function of time-position. This is the only way to make *causality*, which is the determination of a property as an intensive magnitude from its position in objective time, another magnitude. A law that determines one magnitude from another is an algebraic function: the two terms are trapped in an invariable relation. This is the lawfulness over time that we eventually abstract as the semantic category of *causality*.

We have mentioned that Kant’s lists of categories are inconsistent. In the Metaphysical Deduction he lists twelve, and in the Schematism he gives two lists—one containing eight and one containing four. In both cases, the three pure concepts listed under Quantity and Quality collapse into one. In the second case, the three pure concepts listed under Relation collapse into one. This issue will be addressed as I explain the function of each. First, let us look at them in an overview.

Quantity

As for the Quantity and Quality headings, I think Kant’s suggestion that the pure concepts listed beneath them collapse into one makes sense. As we will see, there is a very plausible rendition of the idea that the logical modifiers can control the act of line-drawing and produce awareness of *number*, which is the schema of *magnitude*, or Quantity. The three judgment-forms under Quantity are *singular*, *particular*, and *universal*. We will see that it is the third member under the headings of Quantity and Quality that is the intellectual basis for the semantic value that is named by the heading. The schema of Quantity is *number*, and the objectified rule of Quantity is *totality*, the objective version of the universal quantifier *all*. This is a sub-category of Quantity, along

with *unity* and *plurality*. *Unity* is made under the intention of *this*, the *singular* judgment-form; *plurality*, by intending *some* of the *this*, which is my thinking the rule of the *particular* judgment-form. I then recombine these parts into *totality*, which is my effort of intending the *universal* quantifier. This, is the rule that is schematized by thinking a plurality of unities as a *totality*. This is also the act of *counting*, and its schema, or rule for producing (what would be) images, is *number*. Number is *totality* rendered through time, which is produced by drawing a line. It is what Kant calls a **transcendental time determination**.

It is by making the unity of a line or by operating on that unity that a Kantian pure concept gains its meaning: it makes an instance of this unity in imagination. It is a making, which is the source of meaning. The *Analytic of Principles* is about subsuming. And the concept that subsumes, the meaning or kind of the item it subsumes, is known as the ability to make it—just as *green* means the rule that informs me to “make a hue” and then restricts it to a sub-spectrum of a continuum, which is the topic of the next heading in Kant’s list of tables: Quality.

Quality

Quality is also a heading that contains three judgment-forms: *affirmative*, *negative*, and *infinite*. The third form of Quality is Kant’s addition to the Aristotelian pair of *affirmation* and *denial*, just as the first form of Quantity (*singular*) was Kant’s unorthodox addition to the traditional Aristotelian operators of quantification. The schema of Quality is the awareness of a sense content as a magnitude of sensation, called “intensive magnitude.” This is awareness of a continuum of values, and the awareness that a given (*real*, the sub-category of Quality) has meaning only relative to a continuum of *differences*. *Green* would mean nothing without *non-green*, such as *red*. A person

raised in an all-green world would never grasp *green*, and would never pick out green in intuition, i.e., be able to assert “green” as a true predicate. I *affirm* a given *P* relative to what it is not, but to what it is not *within certain limits*. This additional thought of limitation (in the case of *green*, a limitation in *hue*) is the *infinite* judgment-form, *non-P*.

Non-P is the rule that is schematized as the awareness of *reality* through reference to its *negation*, but in the way of figurative synthesis, or line-drawing. I draw a line in order to produce the sense of magnitude. This is the *non-P*—a continuum of alternate values for *P* that lie in the “same continuum,” or same second-order predicate. In our example, *hue*. The sense of *green* depends on *hue*—*hue* is the logical unity that provides the context that *green* needs to be meaningful. Hue is the infinity of other values that I can produce by knowing the rule *hue*. But here it is taken in a pure way: I am not making these values, but only the space within I will arrange them, as magnitudes, in a way that I can present their difference as a plurality-in-unity. This is the *value continuum*, an a priori condition of kind-awareness, and the category called Quality.

The schema of Quality is that of ascending and descending from the real *P* “in time.” I draw a line, and the distance from *P* as magnitude corresponds to the qualitative difference of a secondary quality, such as *green*. This is a pure concept, so *all possible sensible predicates* must abide by this rule. And Kant manages this by reducing the notion of a continuum of value-differences to the even more universally applicable notion of *intensity*. So, for example, even *chocolate* would be rule for producing a set of instances that differ in a continuum of magnitude. Something can be *more* or *less* chocolaty.

Relation

Here, the disjunctive and hypothetical judgment-forms, I believe, were included simply due to Kant's desire to force his Table of Judgments into conformity with the familiar operators of general logic. This is because they are both ways of comparing two atomic judgments, but not necessary for atomic judgment itself—and thus not truly necessary for sensible cognition. I have already mentioned that the hypothetical judgment-form is actually contained in the subject–predication relation as the algebraic determination of the state-value in P from the time-value referred to by S .

The hypothetical-form is also irrelevant since causality can be judged by means of atomic judgment. As Seung points out, “The causal statement ‘Smoking is a cause of lung cancer’ is not a hypothetical but a categorical statement” (Seung, “Kant's Conception” 116). The disjunctive judgment-form is unnecessary simply because it is irrelevant to the cognition of a physical object and only has meaning in the web of inference. What remains is the subject–predicate relation. This is what we should expect, because only the categorical (or atomic) judgment-form is actually necessary for apperception.

Modality

I skip these entirely because there is no difference between objective modality and logical modality.

THE SCHEMATISM

Schemata generally

Schemata are what give universals their meaning. Universals mean things—intuitions that I can recognize. What I recognize in an intuition is my ability to make it myself. This ability makes use of a rule. The rule tells me how to make any instance that

might ever appear which falls under the “kind” intended by the universal. This “kind” is really my awareness of the rule. I see a green instance, and I know it—as an instance of *green*, the rule for making green images.

A universal is not a particular, yet they clearly relate by some kind of identity. This identity-relation is, Kant says, a procedure. A procedure is, on the one hand, unitary—a procedure. On the other hand, what it makes can be many things. The rule is a *general* procedure. It limits the image to a kind, but in a way that preserves possible variation.

Knowledge is *justified* true belief. I believe by asserting “is P.” *P* is a universal that names a schema. The schema of *P* tells me how to make imaginary instances. If I am aware that one of these would be identical to a given appearance (i.e., something real), then I am aware that “is P” is true. I justify my assertion that something is *P* by comparing the image to the appearance. If they correspond, then I have justified my knowledge.

The rule is the meaning of the universal. Meaning grounds in instances. The universal adds nothing but “the form of generality” [*JL* § 2]. I know the meaning of *green* by producing samples. These are the referents of *green*, and what *green* means. Kant uses an *intuition-based* theory of meaning. It is his analog of the Empiricists’ empirical criterion of meaning.

The official problem that drives the Schematism is a worry over the instances that give meaning to the categories. Empirical concepts have instances that can be “encountered” in intuition *as contents*. But the categories do not refer to empirical contents, they refer to ways of combining point-moments essential to the generic physical object. Ways-of-combination cannot be imagined like empirical contents can be imagined. Ways-of-combination are invisible. Take the computer animation of the

rotating cube again. The temporal identity of the cube, the three-dimensionality of the cube, the fact that “it” is rotating, even the unity of the cube as an outline—none of this is presented in sensibility. “How, then, can an intuition be *subsumed* under a category, and hence how can a category be *applied* to appearances—since surely no one will say that a category (e.g., causality) can also be intuited through senses and is contained in appearances?” [A137/B176].

Kant seems to be returning to the subsumption paradigm that his Copernican method was meant to supersede. But he is not. He is only telling the story of how automatic synthesis is to be carried out with consciousness. It must be possible for the “I think” to accompany all that it is aware of. This spanning of logical, spatial, and temporal difference is carried out automatically, but doing it while asserting “I think” must be possible. This is what we do in judgment—awareness through judgment is the very definition of apperception. The Transcendental Deduction showed us how this is done when we “think” (intend) a line, a circle, and space as three-dimensional. It is done by drawing lines. Then Kant told us that not only spatial objects, but *time itself* is produced by drawing a line, with special attention paid to the fact of passing. Passing, or change, is *created* in act as *movement*:

to make even internal changes [in consciousness] thinkable, we must make time, as the form of inner sense, comprehensible figuratively through a line; and we must make internal change comprehensible through the drawing of this line (i.e., through motion), and hence we must make the successive existence of ourselves in different states comprehensible through outer intuition. [B292]

The premature schematism in the B-Deduction

A transcendental schema (i.e., the schema of a pure concept) is a judgment-form that has been fitted as a rule of figurative synthesis. The synthesis of passing into *time* was accomplished by drawing a line intended as an emulation of passing, but unified by

intending the act of drawing as an effort to “successively determine inner sense, and thereby attend to the succession of this determination in inner sense” [B154]. The passive event of passing is here produced *actively*. A “successive determination” that is given in sensibility is being made through the subject’s activity of “successively determining”—i.e., through drawing a line in a certain way.

There are thus two components of any transcendental schema. First, it must flow from activity. Second, it must be specified—it is not just any act in general. The act, of course, is combination, because it is necessitated by being a unitary subject across logical, spatial, and temporal difference. The specification is enforced by a judgment-form. The judgment-form that produces *time* logical quantification. The act of producing *time* was an act of schematism, but included in the B-Deduction and prior to the Schematism chapter proper.

Now, in the Schematism, Kant revisits the production of *time*, but he now tells us that the concept results from the schematism of the category of **magnitude**, or Quantity. Logical quantification serves as a rule of figurative synthesis, whose schema is **number**, or counting. This is the schema that produces instances of quantity in sensibility. It also produces the instances of space and time. So we now find out that the rule whereby we convert the *form* of space (a way-of-separation) into *formal* space is that of line-drawing in the service of quantification. But now, instead of attending to the act of “successively determining” inner sense, I attend to “the successive addition of one item to another (homogenous item)” [A142/B182]. The determining was *adding* all along, but now this is made clear.

This attention to adding while line-drawing, Kant says, is what lets me constitute the result of my act as a “conjoint” successive addition. This produces, along the way, space and time themselves. We now find out that space and time are actually *images*

produced by me—pure images: “The pure image of all magnitudes (*quanta*) for outer sense is space, whereas the pure image of the magnitudes of all sense objects as such is time” [A142/B182]. Space and time are images, produced by line-drawing, with attention paid to accumulation-through-addition.

Formal space is magnitude: intuiting unity depends on judging unity

In the B-Deduction we discussed the difference between the *form* of outer sense (which is innate) and *formal intuition*, which is space as unified field. We have described it as a first synthesis that both combines the plurality of sense-consciousnesses into a unitary “I think” and also combines the plurality of sense data into the field of space. This synthesis is carried out by line-drawing, and results in what we have called the intuiting unity of epistemic consciousness. The unity of the line—of the “I think” and of space—derives from the *identity of the activity*. For Kant, activity (spontaneity) is the primordial source of all unity. Every point of the line is produced by *one and the same act*. This identity makes the points in the line *members* of one unitary line; the line is a plurality-in-unity. This identity also combines the plurality of sense-consciousnesses attached to each point into one unitary “I think”; the “I think” is a plurality-in-unity.

For Kant, formal space cannot be an object *for judgment* until it has been cognized as an extensive magnitude. Doing this *involves a rule of judgment*. Kant *treats* the intuiting unity of space in the Transcendental Aesthetic, the part of the First *Critique* that is supposed to be occupied solely with the faculty of intuition. But we find out, as Kant later admits, that everything therein is actually dependent on synthesis—not in some generic sense, but in particular on *logical quantification*, applied in ostensive judgment. The formal space constructed through line-drawing is the very “image” of magnitude: “The pure image of all magnitudes (*quanta*) for outer sense is space” [A142/B182].

Kant's proviso "for outer sense" means *for reality* and *of real presentation*. The result of threefold synthesis (apprehension, reproduction, and recognition of a line) being overlaid on outer sense is the *image* of magnitude, i.e., its particular.

Formal time is also spatial magnitude

The next problem is the cognition of *time*. Time has significance for the subject because it is originally generated through the act of line-drawing: "For as regards time, which after all is not an object of outer intuition at all, we cannot present it to ourselves except under the image of a line insofar as we draw that line; without exhibiting time in this way, we could not cognize the singleness of its dimension" [B156].

The same act of line-drawing that produced space also produces—*originally* produces—time. For Kant, time is *entirely* a product of imaginary construction. This is because time itself cannot be really presented. Time is just the imaginary combination of the "dimension" of *passing*, and the passing-away of presentations cannot itself be presented as a presentation. Only connections among simultaneous and compresent data can be presented in intuition—that is, only *spatial* relations can be presented. Outer sense contains a priori elements (proto-space) *and* can display them, once we draw lines through outer sense in order to apprehend these pure and homogeneous elements. But the passing-away of the empirical contents of the spatial field cannot be presented. The "relation" of passing-away cannot be presented because the contents that we want to relate are absent.

In the case of space, imaginary space is *identical* to "real" space. (When I trace a given triangular pattern of point-data, my imaginary construction is *isomorphic* with a "real" pattern given in sensibility.) But there is no "real" presentation of time. The power

of intuition is exhausted by outer sense. Time is not presentable *as* time (as *passing*), but rather generated *originally* as space. *The science of time turns out to be geometry:*

We present time sequence by a line progressing *ad infinitum*, a line in which the manifold constitutes a series of only one dimension. And from the properties of that line we infer all the properties of time, except for the one difference that the parts of the line are simultaneous whereas the parts of time are always sequential. This fact, moreover, that all relations of time can be expressed by means of outer intuition, shows that the presentation of time is itself intuition. [A33/B50]

But this presents a problem: since both space and time are constructed through the same act of line-drawing, how can we account for their distinction? The answer is that the sense of the line-drawing activity is modifiable *according to the attention of the subject*:

And even time we cannot present except inasmuch as, in *drawing* a straight line (meant to be the externally figurative presentation of time), we attend merely to the act of the manifold's synthesis whereby we successively determine inner sense, and thereby attend to the succession of this determination in inner sense. [B154]

So by drawing a line *while paying attention to* the “succession” of inner sense (internal passing) that is occurring within me throughout the my activity of line-drawing, the sense of the line that I draw becomes temporal, instead of spatial. Time is produced by the activity of line-drawing, augmented by focusing on *succession* during the act.

Schema of magnitude

The *pure schema of magnitude (quantitas)* taken as a [pure] concept of understanding is *number*, which is a presentation encompassing conjointly the successive addition of one item to another (homogeneous item). Therefore number is nothing other than the unity in the synthesis of the manifold of a homogeneous intuition as such, a unity that arises because I myself produce time in apprehending the intuition. [A143/B182]

Space and time are pure images of magnitude. When magnitude is presented, it takes these pure images as its *matter*. Every outer magnitude is a sub-space, and every inner magnitude is a sub-time (understood as a sub-space, since time inherits the structure

of space). I create unified (formal) space in dependence on the time-taking act of line-drawing, and I present the fact of passing in space through movement, and present the accumulation of passing moments in the line's spatial extension. But the schema of magnitude is neither space nor time, but a grammatically interpreted procedure of line-drawing, called *number* (or counting). In this case, I draw a line while attending to my act of extending "it." That is, I think the line I have made as a unity by thinking it under the *singular* judgment-form "this," which grammatically intends the sub-category of *unity*. Then I think this unity only in part, by thinking it under the *particular* judgment-form "some." This divides the line into parts, into an awareness under the sub-category *plurality*. Finally, I totalize these parts, or *count* them, by thinking it under the *universal* judgment-form "all"—and this produces awareness under the sub-category *totality*. This threefold process is *counting*, and this "presentation of a method for presenting" a totalized plurality of units in an image—its schema—is *number*.

Remember that Kant sees the third element under a heading to be the combination of the previous two. Here, the *universal* judgment-form is a combination of the *singular* and *particular* forms, so the sub-category of *totality* is a combination of *unity* and *plurality*—it is a plurality of units that is itself unified. The category of Quantity arises from the third element under the heading, *totality*, which thinks "all" of the parts into a whole, as it is applied to time, or line-drawing. The schema needs all three logical operators to perform a figurative function.

The third sub-category of Quantity, *totality*, is in fact synonymous with the heading, Quantity. As Kant explains in the Metaphysical Deduction, the third in the set results from the combination of the previous two. This lets us understand the relationship between the heading and its three sub-categories. The first two combine to enable the third, which is synonymous with the heading. We will see that the third element under the

Quality heading is also responsible for guiding that category's schematism. This does not hold, however, for the second pair of headings, Relation and Modality.

The transcendental schema of Quantity (or extensive magnitude) is the *temporal rendering of totality*. Passing is the maximal container of plurality. To apperceive via "This (S) is P," I must span across it according to each of the judgment-forms of intellectual synthesis. Intellectual synthesis becomes figurative synthesis when it is rendered temporally. *Totality* is the discursive rule that is schematized, or made fit for time, by drawing a line while producing *number*. Doing this creates the semantic content of *magnitude*, which applies to the sense-world a priori because (formal) space and time themselves are *products* of the rule that turns *this* line into *parts* that are then recombined into a *whole* that is a plurality-in-unity or *number*.

Note that in the Metaphysical Deduction Kant says that logical quantification in transcendental logic belongs to the subject-position. This accords with his claim in the Metaphysical Deduction that quantity is thought "in" the subject-position. [A71/B96] The magnitudes are *body* and *substance*. The former is the magnitude of my (the point's) identity as I change position in an active emulation of passive change—a line as a spatial span. The latter is the magnitude of my (the point's) identity as I perdure as act in the face of the passing of what I receive—substance as a temporal span. What "spans" in these cases is *reality*. The body is reality *extended* through space as a homogeneous reality. Both products result from how a way-of-separation becomes a substratum of *real identity*. *Body* and *substance* are objective reflections of the identity of my act across their respective ways-of-separation (space and time, respectively).

Schema of reality

Kant should have called this the schema of Quality, since what is being produced is an entire continuum of qualitative values, and not just one. Instead, he here identifies the heading with the first sub-category, *reality*. Reality is the content of a sensation rendered as a magnitude, called *intensive magnitude*. This magnitude is what empirically “fills” a point-moment. This filling is variable—it can be filled “more or less.” It can be the magnitude of what is now before me in the appearance. This is the meaning of *reality*. But it could have been, and might be, “less” this way. I receive a red appearance of intensity n , but I know a priori that it could have been *not* this intensity. But I really know more than this—I know that it could have been different but *must* have been a color. Knowing this background continuum of possible other colors is the Quality of which a particular red is an instance. This is Kant’s mathematical model of kind-recognition, or subsumption by predication. “Hence there is a relation and coherence, or rather a transition from reality to negation, which is responsible for every reality’s being presented as a quantum” [A143/B182].

“And the schema of a reality taken as the quantity of something insofar as it fills time in precisely this continuous and uniform production of that reality in time, where from a sensation having a certain degree we descend, in time, until the sensation vanishes, or ascend gradually from the sensation’s negation to its [actual] magnitude” [A143/B182–83]. This is really the schema of Quality, whose third sub-category is *limitation*. *Limitation* is the limiting power of the rule of image-making. I limit my production to a range of shades of red, or a range of hues. Unfortunately, Kant seems to limit this continuum to one of intensity only. But this would be a misreading.

I am sensibly stimulated so that I receive or am presented with a sensation of having a particular (real) value (intensity) of n ($r = n$). What could it mean for this value

n to descend until the sensation vanishes? What would $r = 0$ look like? If it means that the red becomes transparent, what could *this* mean? Transparent would seem to be the proper descriptor for $r = 0$. But there is no such thing as no-color. The transition from red to black (brightness = 0) results in a positive value—black. The transition from red to white (saturation = 0) also results in a positive value—white. The transition to another hue results in a different color—orange or violet.

I suggest replacing Kant’s official notion of a descent from $r = n$ to $r = 0$ with a different notion—one that describes the actual function of Kant’s mathematical conception of Quality in experience. The schema should simply be the notion of a *continuum of values within some second-order type*.

What is being schematized here is reality “taken as the quantity of something insofar as it fills time in precisely this continuous and uniform production of that reality in time.” The “time” here is a time-position in the time-order (subjective or objective). The reality that fills it does so in a “continuous and uniform way”—Kant means that when we schematize a given, real intensity, we are drawing a line, which is the only means by which we can generate the sense “continuous magnitude,” or *quantum*. Of course, the sense-content before me is not a line, but I *imagine* it as a value. By my model as a value of hue, saturation, brightness, etc. The empirical content is given; but the notion of content-as-value, a notion that is necessary for the practice of mechanics, is parasitic on the act of line-drawing, which constructs the continuum of state-values by “descending” or “ascending” in time.

The schema is a rule modifying the act of line-drawing—we descend or ascend *in time*. Again, the crucial temporal element of the act (its successive nature, which is tied to following a procedure, or rule) becomes schematized spatially, as spatial distance. The

distance here is intensive, but it is rendered intelligible only as spatial distance. I can ask how “far” the current magnitude of reality is “from” zero.

Schema of substance/property

The schema of substance is the presentation of the real as a substratum of empirical time determination as such, a substratum which therefore endures which all else varies. (Time is not in transition; rather, the existence of what is mutable is in transition in time. Hence to time, which is itself immutable and enduring, there corresponds in [the realm of] appearance what is immutable in existence, i.e., substance; and only by reference to substance can succession and simultaneity of appearances be determined in terms of time). [A144/B183]

Property is not mentioned, but it corresponds to that which varies, the content that passes away.

Substance is the universal whose rule is the subject-position when it aims towards the passing pixels and thinks an object. We already know that the real fills spatial magnitude as *body*, which is also the referent of the subject-position.

The apriority of grammatical subsumption explained

In the Metaphysical Deduction, Kant notes that one of the consequences of abstracting from all content of cognition is that “the understanding’s merely logical use left undetermined to which of the two concepts we want to give the function of the subject, and to which the function of the predicate. For we can also say, Something divisible is a body” [A94/B128–29]. In logical space, we can subordinate body under divisible (All bodies are divisible) or we can do the reverse (Some divisible things are bodies). In non-ostensive judgment, the subordination relation is reversible. But in transcendental logic, and thus ostensive judgment, the subject term automatically subsumes the perduring body of the transcendental object. The grammatical roles of ostensive judgment are irreversible because their sensible referents are extra-logically different.

In general logic, subject and predicate contents are concepts, the only difference being that the former is the “condition” of the latter. Logical roles alone cannot indicate the proper assignment of universals. As Kant shows in his example, the concepts in non-ostensive judgment can be reversed and the logical relation preserved simply by changing the proper logical operators.

But in transcendental logic the subject and predicate contents are not concepts, but transcendental content in intuition, i.e., aspects of sensible synthesis. In ostensive judgment, the subject-position has the sensible-synthetic function of constructing the spatiotemporal version of the grammatical subject—a spatiotemporal subject, which we abstract as *body* and *substance*. The result is a cognition of real spatial coherence (body) and real temporal continuity (perdurance).

Now we can understand the meaning of this passage from the Metaphysical Deduction: “If, on the other hand, I bring the concept of a body under the category of substance, then through this category is determined the fact that the body’s empirical intuition in experience must be considered always as subject only, never as mere predicate” [A95/B129]. What can “never” be a predicate is the kind of sensible synthesis produced by the ostensive subject, which is a rule of synthesis. And because substance is just the reflection of this determination of sensible synthesis, assigning body to substance also assigns it to the aspect of sensible synthesis from which substance was reflected. And because this aspect is produced (albeit blindly) by the effort at finding a candidate for the ostensive subject-position, it is also automatically apprehended and reproduced under that position. Even if it is not recognized as a universal, or even as a constructive procedure, its subsumption under the subject-position occurs a priori.

A grammatical position automatically subsumes the aspect of sensible synthesis which it blindly constructed because of the mere effort at thinking appearances in a way

conformable to judgment. When the savage points at the house, he is already tying the house as substance (perduring body) to the subject-position, even though his concept of substance has not been consciously developed.

Grammatical positions in ostensive judgment are irreversible because their sensible referents are unique in a way that the logical relation of condition–conditioned cannot be. The aspects of sensible synthesis which these positions subsume are permanent assignments, tied to these positions by the rules that produced them. Consequently, any semantic universals assigned to these positions cannot be reversed once an assignment has been made by, for example, subsuming a candidate universal under one of the semantic categories. When the transcendental power of judgment recognizes, the “significance” of subsuming an aspect of sensible synthesis can be nothing other than threefold synthesis interpreted into a form of judgment. It is the form of judgment that adds the particularizing sense of its own “logical signification.” This logical sense then seeks-out what is homogeneous with itself, so it subsumes an analogous unity in space and time. This mapping, however, is done blindly.

Schema of causality

“The schema of the cause and of the causality of a thing as such is the real upon which, whenever it is posited, something else always follows. Hence this schema consists in the manifold’s succession insofar as this is subject to a rule” [A144/B183]. Every content is what it is by law. Contents change, so the value of a content is determined a priori according to its position in time. The value of a content follows from a “rule,” and the rule relates contents to each other in time as having the values they do by virtue of their *time-position*. The *hue* of this point here is what it is because the time is *now*. Now determines the value of the predicate.

Both time and Quality are magnitudes. Time is extensive, and is the referent of the subject-position. I span space and time and create a trans-spatial and trans-temporal referent for my innate *this*. This is the intentionality of the subject-position. Now, I can focus on the body-aspect of this object and make a mathematical judgment about its figure. Or I can focus on the substance-aspect of this object and consider the transience of a *property*. Then I am aware that every content is positioned in time. This relation says that the content at a point-moment is determined to be that content as a function of its time-position, which is a magnitude. And the content—it is a value whose meaning is precisely its differential relation to the non-*P* judgment-form, figured as a continuum of, say, *hue*, taken as the totality of non-*P*s belonging to the same kind as *P*, and making *P* meaningful-by-difference. In an all-red world we would sense red but never think it, i.e., never subsume it as *P* via “is *P*.” This way-of-difference is “intensive”—I can move away from the red part of the hue-spectrum in a way that is incremental content-wise. This difference is logical—*red* and *orange* are distinct. But it is also magnitudinal—*orange* is less *non-red* than *yellow* is.

Therefore, the subject–predicate relation in ostensive judgment is a relation of time-magnitude to property-magnitude. This, Kant says, is the schema of causality. But it is a unity effected through the *copula*, “is”—there is no hypothetical judgment involved. This pixel *is* this content because of now. This relation is a determinative relation between magnitudes: the value of one magnitude determines another, by a rule or law. But this can itself only be a mathematical transformation on the domain of real numbers. Time is a line, and every time-position is a value on a continuum. The subject-position is thus entirely isomorphic with an algebraic variable ranging over the set of real numbers; or, if graphing, over an **axis** of Cartesian coordinate space. The predicate-position likewise is an algebraic variable ranging over the continuum of *intensive* magnitude. Kant

has said that variation within a secondary quality occurs infinitesimally, and we also find out that there are no leaps in change.⁶²

Causality: continuous change in continuous time, under a single rule

Now every change has a cause that manifests its causality in the entire time wherein the change takes place ...; so that, as the time increases from its initial instant (*a*) up to its completion (in *b*), the reality's magnitude (*b – a*) is also produced through all the smaller degrees contained between the first degree and the last. Hence all change is possible only through a continuous action of the causality. [A208/B253–4]

This clearly states that one magnitude, *reality* (Kant's misnomer for Quality),⁶³ is “produced” by another, *time-position*, or magnitude of the real substratum. Causality is the bridge between substance and Quality.

Moreover, this change is continuous—both time and Quality vary continuously: “This, then, is the law of the continuity of all change. The basis of this law is this fact: that neither time nor, for that matter, appearance in time consists of parts that are the smallest; and that nonetheless, as a thing changes, its state passes through all these parts, as elements, to the thing's second state” [A209/B254]. Kant is imposing the continuity of space into the realm of sensible change by making time and space through line-drawing, as continuous magnitudes, both presented as lines, but interpreted differently: one is the continuum of time itself (substance), and the other the continuum of possible state-values (Quality).

This is just an algebraic relation between two real numbers, one determined both in a relation of continuous variation. As time-position changes by the amount ($S_2 - S_1$),

⁶² After the Refutation of Idealism, Kant writes: “But all four propositions [*in mundo non datur hiatus, non datur saltus, non datur casus, non datur fatum*] unite in this: that they admit in empirical synthesis nothing that could impair or interfere with the understanding and the continuous coherence of all appearances, i.e., the unity of understanding's concepts. For in understanding alone does the unity of experience, the unity in which all perceptions must have their position, become possible” [A229–30/B282]

⁶³ See the “Quality” section above.

state-value changes by the amount $(P2 - P1)$. The predicate is thus an algebraic variable that ranges over the continuum of real numbers, and whose particular value that changes continuously as an algebraic function of the subject variable, which is the independent variable of the function. For every value of S , the value of P is determined by some function.

Quality is also an axis in Cartesian space—an orthogonal axis since its variation is that of what fills time at every moment, and value that ranges over an independent degree of freedom: variability in the content of a sensation. The result is two magnitudes that relate as two orthogonal axes, and the particular law is the algebraic function that produces the smooth, differentiable curve in Euclidean space. “This (S) is P” has as its pure structure “ $t \rightarrow f(t)$.”

Time is continuous, due to the space within which passing is presented for figurative synthesis. And change, also, is continuous—change occurs continuously, without leaps. This demand for continuity is nothing other than the identity of apperception, as it manifests when I combine the ways-of-separation—logical, spatial, and temporal. All three are structured spatially—qualitative (logical) difference is unified in a continuum of infinitesimal differences, space is a continuum due to my identity moving through it in line-drawing, and time is a continuum of passing that I emulate by moving a point. These continuities are all figured spatially—as the continuity of space. And all three are continuums over which the subject and predicate-positions range as algebraic variables. The unity of the variable is the unity of apperception, the unity of the infinite continuum of a line under a single “thought.” The figurative expression of the principle of apperception is the principle of continuity. This principle prohibits “in the series of appearances (changes) any leap (*in mundo non datur saltus*); but it also

prohibit[s] in the sum of all empirical intuitions in space any gap or breach between two appearances (*non datur hiatus*)” [A228–9/B281].

Kant assigns a different principle of continuity to each of the headings in his tables: “We could easily present these four propositions (*in mundo non datur hiatus, non datur saltus, non datur casus, non datur fatum*), like all principles of transcendental origin, in their order, according to the order of the categories, and assign to each its position” [A229/B282]. There is no “break” in Quantity (in time or space, the images of magnitude). There is no “leap” in Quality (in the change from temporally contiguous predicate-values). There is no “accident” (every state-value is determined by its position in time, or time-value, by an algebraic law that transforms time-value into the state-value really given). And finally, there is no “fate,” since this mathematical function operates on the domain of appearances, and not on the noumenal stimulator outside me. All four propositions

unite in this: that they admit in empirical synthesis nothing that could impair or interfere with the understanding and the continuous coherence of all appearances, i.e., the unity of understanding’s concepts. For in understanding alone does the unity of experience, the unity in which all perceptions must have their position, become possible. [A229–30/B282]

The “unity” of the pure concepts is the unity of the continuum.

The causal nature of the copula

As Longuenesse points out, Kant identifies the copula as the source of the causal relation, and not the hypothetical judgment-form, in the *Prolegomena*.⁶⁴ There, Kant is discussing the process that upgrades “judgments of perception” (which refer to subjectively valid unity) to “judgments of experience.” He then gives an example of how

⁶⁴ Longuenesse, *Capacity* 175.

this is done for *Air is elastic*. It is done by subsuming *air*, the referent of the subject-position, under the category of *causality*.

Judgments of experience are of quite a different nature. What experience teaches me under certain circumstances, it must always teach me and everybody; and its validity is not limited to the subject nor to its state at a particular time. Hence I pronounce all such judgments as being objectively valid. For instance, when I say the air is elastic, this judgment is as yet a judgment of perception only—I do nothing but refer two of my sensations to one another. But, if I would have it called a judgment of experience, I require this connection to stand under a condition, which makes it universally valid. I desire therefore that I and everybody else should always connect necessarily the same perceptions under the same circumstances.

Quite another judgment therefore is required before perception can become experience. The given intuition must be subsumed under a concept, which determines the form of judging in general relatively to the intuition, connects its empirical consciousness in consciousness generally, and thereby procures universal validity for empirical judgments. A concept of this nature is a pure a priori concept of the Understanding, which does nothing but determine for an intuition the general way in which it can be used for judgments. Let the concept be that of cause; then it determines the intuition which is subsumed under it, e.g., that of air, relative to judgments in general, viz., the concept of air serves with regard to its expansion in the relation of antecedent to consequent in a hypothetical judgment. The concept of cause accordingly is a pure concept of the understanding, which is totally disparate from all possible perception, and only serves to determine the representation subsumed under it, relatively to judgments in general, and so to make a universally valid judgment possible.

Before, therefore, a judgment of perception can become a judgment of experience, it is requisite that the perception should be subsumed under some such a concept of the understanding; for instance, air ranks under the concept of causes, which determines our judgment about it in regard to its expansion as hypothetical.* Thereby the expansion of the air is represented not as merely belonging to the perception of the air in my present state or in several states of mine, or in the state of perception of others, but as belonging to it necessarily. The judgment, “the air is elastic,” becomes universally valid, and a judgment of experience, only by certain judgments preceding it, which subsume the intuition of air under the concept of cause and effect: and they thereby determine the perceptions not merely as regards one another in me, but relatively to the form of judging in general, which is here hypothetical, and in this way they render the empirical judgment universally valid. [*P* § 20]

Thinking the rule that ties a quality-continuum to a substance (time-continuum) is how I make the judgment objective, and how I cognize what Kant calls *objective time*, the time of intersubjective agreement. But intersubjective agreement is just necessity: what we agree about is what *must* be the case. Something about the sensible object of judgment is the way it is by necessity—this is the topic of universal agreement. What is necessary is, first, its having a position in time and extension in space, which are universally shared *objective* forms. Second, it is necessary that its state-change be presented in a *certain* order in time. Time-order of contents that differ by intensive magnitude is pre-determined, and determined by a single rule, or law, that can be discovered. The intensive magnitude in the predicate-position is determined by the temporal magnitude in the subject-position.

An even clearer example of the causal role of the subject–predicate relation in figurative synthesis is given in the footnote in the passage above:

An easier example is: ‘When the sun shines on the stone, it grows warm.’ This is a mere judgment of perception and contains no necessity, no matter how often I and others may have perceived this. But if I say ‘The sun warms the stone’, which means that the sun *causes* the stone to become warm, the concept of *cause* is added to the perception and connects the concept of warmth necessarily with the concept of sunshine. [P § 20 fn. 12]

Precedent for the causal copula exists in Wolff

For Christian Wolff, all synthetic categorical judgments are covert hypothetical judgments. This is a legacy from Leibniz, who saw all truths as necessary and ultimately analytic, so that their syntheticity is really obscured analyticity, and could be discovered by analyzing the subject and predicate concepts into their simples. As Longuenesse

points out, while Lambert and Meier⁶⁵ take the hypothetical proposition as a species of composite propositions,

Wolff studies it together with the categorical proposition as one of the two possible relations of the assertion to its condition, and consequently discusses it before introducing the distinction between simple and composite propositions. Kant retains precisely this aspect of the relation of the assertion to its condition as the common heading of the three *relations* or *exponents* of a judgment. ... Kant is the first to bring together hypothetical, disjunctive, and *categorical judgments* under the heading of relation (of the assertion to its condition). (Longuenesse, *Capacity* 98 fn.)

The difference between the categorical and hypothetical forms is that a categorical proposition is one “in which the predicate is stated about the subject absolutely, or without any added condition,” while a hypothetical proposition is one “in which the predicate is attributed to the subject under an added condition.” This is the meaning of the ellipsis in “if ... then”—something is *consciously* added. The categorical form is absolute, the predicate (state-value) is determined by the subject (time-value) *alone*. This relation, like the hypothetical judgment-form, is still one of condition—conditioned, but nothing *needs* to be added: it flows from the fact that the subject-position binds (space and) time, while the predicate-position binds Quality as a magnitude.

“Wolff argues that every categorical proposition can be formulated as a hypothetical by making explicit the relation of the assertion to its condition” (Longuenesse, *Capacity* 100).⁶⁶ Kant does just this, and expands the time–state relation

⁶⁵ See § 131 of Johann Heinrich Lambert, *Neues Organon, oder Gedanken fiber die Erforschung und Bezeichnung des Wahren und dessen Unterscheidung vom Irrtum and Schein*, 2 vols. (Leipzig, 1764); and § 304 of Meier, G.W.F., *Auszug aus der Vernunftlehre* (Halle, 1752), in Kant, Ak. XVI.

⁶⁶ The proposition “A regular figure can be inscribed in a circle,” Wolff says, is equivalent to “If a plane figure is equilateral, with equal sides, it can be inscribed in a circle.” Similarly, “God created the world” is equivalent to “If God is the most perfect being, then He created the world.” See § 226 of Christian Wolff, *Philosophia rationalis sive Logica* (Frankfurt and Leipzig, 1740), in Wolff, *Gesammelte Werke*, II-1, 3 vols.

contained in the categorical form into the hypothetical judgment-form *itself* and by entering it into his Table of Judgments as the basis of the pure concept constructing physical causality. But lawfulness, as the objective (necessary) time-determination of appearances (as state-values) belongs originally to the copula. As we noted earlier, for Kant

a judgment is nothing but a way of bringing given cognitions to the objective unity of apperception. This is what the little relational word *is* in judgments intends [to indicate], in order to distinguish the objective unity of given presentations from the subjective one. [B141–42]

Objective unity is objective *time-order*, subjective unity is the subjective time-order by which I happen to apprehend something. Contents may be apprehended in many ways; but contents are values that are determined by time-position, and this is the meaning of objective time order.

THE SYSTEM OF ALL PRINCIPLES OF PURE UNDERSTANDING

Chapter II of the *Analytic of Principles* is the most important in the First *Critique* goal-wise since it is where Kant draws-out the objective implications of the effects of transcendental synthesis.

Kant opens Chapter II, the chapter in the *Analytic of Principles* coming after the *Schematism*, with a discussion of “universal principles,” i.e., ones that hold of objects of knowledge in a domain of difference. There are three sections. In the first two, Kant announces simple universal principles, i.e., ones that hold of objects of knowledge in a domain of difference—the logical domain of difference (the domain of already-abstracted universals) and the spatiotemporal domain of difference (the domain of passing pixels).

Section I: the principle of analytic necessity

The first is the “supreme principle of all analytic judgments,” i.e., the principle of noncontradiction. This tells us nothing positive about the object, only that our judgments about it cannot contradict themselves. It is a law of thought: a contradiction “annihilates and annuls” any content of judgment. It is also the principle for cognizing analytic truth. The necessary truth of analytic judgment rests on the fact that an analytic predicate *must* be affirmed of the object, otherwise a contradiction would arise and thought itself would cease.

Section II: the principle of synthetic necessity

The second is the “supreme principle of all synthetic judgments.” The possibility of synthetic judgments a priori is the official problem of the entire First *Critique*. How can a non-analytic judgment enjoy necessary truth? This is how Kant tackles Hume’s skepticism, which limits necessary truth to analytic judgments alone. This renders unwarranted the necessary truth of all judgments that are non-analytic, including ones that we all know *are* necessarily true. These are truths that follow from the categories having objective reality. The categories of *magnitude*, *substance*, *quality-as-magnitude*, and *causality* are *real*. To say that *substance* has objective reality, or “Substance is real,” implies a necessary truth about objects of knowledge, i.e., that “The real is permanent, meaning that its quantity is conserved.” As a consequence of categories being necessarily applicable *and* objectively real entails that certain necessary truths about objective reality now hold. When I span space and time by moving a point under the rule of the subject-position I am conscious of *reality* spanning space and time—as a **body** and as a **substance**, respectively. When I span the continuum of values thought in the kind of the predicate, I am conscious of reality as a value on a continuum, as an intensive magnitude or **quality**, that varies infinitesimally over time. Finally, reality is lawful: the value of

intensive magnitude is determined as a function of time-position—one magnitude is determined, or calculated, as a function of the other. This is what I think through the copula: the value of *S* is the *condition* of the value of *P*, where *S* ranges over the values of the objective time-continuum. Being conscious of this mathematical predetermination, I am conscious of state-values being determined **causally**.

These principles are the consequence of the supreme principle of synthetic judgments, i.e., that the ways-of-separation provided by sensibility be combined, or spanned-across, by unitary apperception, which is aware as “I think that this (*S*) is *P*.” As we have said, Kant reduces the two forms of intuition to one—time, the maximal container. This necessity, the synthesis of time under judgment-forms, is the one principle shared in common by each of the physical principles that Kant will now discuss. The *conscious* synthesis of the passing way-of-separation under the rules of the various judgment-forms, now acting as time-schemata, is the “third” that binds the subject and predicate in synthetic a priori judgment. Analytic judgment needs no third except the principle of noncontradiction. The “third” of synthetic (ostensive) judgments is the *object* itself, which is objectified unitary apperception, an objective “This *S* is *P*.”

All passing pixels are combined in imagination so as to provide objects of apperception, or consciousness-through-judgment. Without this synthesis, there could be no object that is *P*, and thus no objective epistemic awareness, and thus no *awareness*:

experience would not even be cognition, but would be a rhapsody of perceptions. Such a rhapsody of perceptions would not fit together in any context conforming to rules of a thoroughly connected (possible) consciousness, and hence would also not fit together to agree with the transcendental and necessary unity of apperception. Hence at the basis of experience there lie, a priori, principles of its form. [A156/B195–96].

We already have an idea of how such productive cognition occurs through the familiar example of drawing geometrical figures. I make a figure all by myself, using a

priori space and positing a trans-spatially identical point, my unity, through space as movement, thereby apprehending it under one act, and so unifying it for one consciousness. Now Kant points out that this makes geometrical knowledge objectively *real*. Real objects, ones I automatically synthesize from appearances into physical objects, conform to the same spatial framework that I draw-through in my a priori acts of geometrical construction. The laws of geometry and the facts of arithmetic thus hold of real objects. The same is true of my *other* acts of figurative synthesis. When I draw a line while attending to an identical, moving point, I make time as a magnitude *and* as a substrate. Reality spans through time because my act is reality-making and it perdures through time as a constant. The time through which I act myself is the same time in which appearances are forced upon me. So the reality that I make by moving an identical point is also real. Just as mathematical objects have objective reality due to being constructions in reality-receiving space, so also the figurative syntheses under grammatical intentions have objective reality due to being constructions in reality-receiving *time*.

Thus synthetic judgments are possible a priori if we refer the formal conditions of a priori intuition, the synthesis of imagination, and the necessary unity of this synthesis in a transcendental apperception to a possible experiential cognition as such, and if we then say that the conditions for the *possibility of experience* as such are simultaneously conditions for the *possibility of objects of experience* and hence have objective validity in a synthetic a priori judgment. [A158/B197]

This, finally, explains a priori truths about (physical) objects. Spanning ways-of-separation (time, but also space since time is produced spatially) so that I can be aware of “S is P” in the sequence of passing pixel arrays, an act that is a necessary condition of awareness (conscious awareness through judgment), since it is a condition of objective awareness, is also a condition of the awareness of objects, and thus of *objects themselves*

universally. This is Kant's Copernican method, which will now yield principles of physical existence

Section III: the Systematic Presentation of All Synthetic Principles

The Systematic Presentation lists Kant's principles of physical existence, necessary truths about the generic physical object. First and most obviously are the principles of *mathematics*. These will not be included individually, because they fall under a higher principle called the Axioms of Intuition. The principles of pure intuitions are extrapolations from the form of intuition, and do not describe a pure *concept*, or rule. The rule is line-drawing while adding the fact of succession, which I make through act as movement. Only the act of drawing *itself* is guided by an a priori rule. I am forced to unify space as magnitude, but I am not forced to generate this or that figure.

The principles of pure Euclidean space hold of real objects, since reality is received by me according to the spatial way (form) of separation. Space, as we have seen, is unified under the rule of *magnitude*, whose schema is *number*. Space and all sub-spaces arise as outer images of magnitude. Space is always already a magnitude, and space is how I receive reality, so reality *qua* spatial inherits all the apriority of geometry. The same hold of arithmetic: I count objects just as I count tick-marks. Both are temporal aggregates made by adding-in-time.

Mathematical and dynamical principles

Kant divides the principles into two types: mathematical and dynamical. The principles derived from the categories under the Quantity and Quality headings are called mathematical principles because they justify applying mathematics to appearances and treat individual intuitions and their objects as quantifiable magnitudes. The principles derived from the categories under the Relation and Modality headings are called

dynamical principles, Kant says, because they treat the relation of appearances to “existence,” i.e., to the fact of their being *given* to sensibility. Figures in space and numbers of things do not include passing in their sense. A triangle is an object in a time-slice. When I think it, I think it in spatial terms alone. Numbers, while gaining their sense from the time-taking process of counting, are also atemporal in their sense. But a datum being filled by extra-volitional stimulation makes direct reference to the *real given*, which passes ways, and so acts on me continually in time. This is the “dynamic” of Kantian existence.

There is another difference between the two types of principle. Mathematical principles deal with intuition alone. Because mathematical objects are directly present in intuition (as line, figure, or number), mathematical truths are transparently necessary: I see the truth of a mathematical judgment in an object that is directly presentable in intuition as an image. But the dynamical principles cannot be presented directly in intuition. The meaning of *substance*, *property*, and *causality* is not abstracted from an image, but from an interpreted activity, i.e., that of line-drawing. And what the act of drawing emulates, or schematizes, is the fact of *real passing*. In this way, the dynamical principles are contingent—I must be stimulated before their truth can be verified. These principles deal with how I think and imagine point-moments as being trans-temporally connected. I can present this process by line-drawing, but its sense is *not* presented in the product of that act, as are Quantity and Quality, which are magnitudes whose images are space and time (imaged as a line).

Magnitude (extensive): Axioms of Intuition

The forms of sensible plurality are ideal, but they contain reality

The principles resulting from the objective reality of the category of *magnitude*, rendered as *referring* to space and time themselves, are called the Axioms of Intuition. All magnitude gains its sense from spatial (temporal) extension. But I need not *refer* magnitude to spatial extension. I could refer to the quality of a sensation as a magnitude: “Austin is hotter than Miami.” But here, Kant is referring to the direct overlay of mathematical constructions onto the receptive framework that delivers the sensible contents stimulated by reality.

Kant rewrote the introductory definitions of each of the principles. I will list them separately:

- A Edition: “*Principle of pure understanding*: All appearances are, in terms of their intuition, extensive magnitudes” [A162].
- B Edition: “Their principle is: *All intuitions are extensive magnitudes*” [B202].

The spatial and temporal situation of *physical objects* inherit all the apriority of geometry and mathematics. The physical world *itself* is quantifiable, because its matter, the filled point-moments of sensibility, can only appear to me through my innate forms of pluralization. These are nothing to me until my possible “I think” manifests its force by positing and moving a point. As I accumulate passing and positions, I invent space and time—as magnitudes. This same space and time is the one that contains all the sensible reality I am aware of. Sensible objects are made of data that are pre-situated and pre-interrelated by the framework that I construct independently of being stimulated. The space I assemble through line-drawing is the space of appearance and therewith reality.

Magnitude as extension

Extensive magnitude is a *kind* of magnitude. Remember that the categories are ways-of-unity that flow from judgment-forms, which apply to sensible intuition “as such,” as well as to universals. This kind of magnitude is completely unspecified. Now, when it is being applied to space (and time), it gains intuitive significance—as *extensive* magnitude. This is the magnitude produced by moving a point. And so Kant defines extensive magnitude as

a magnitude wherein the presentation of the parts makes possible (and hence necessarily precedes) the presentation of the whole. I can present no line, no matter how small, without drawing it in thought, i.e., without producing from one point onward all the parts little by little and thereby tracing this intuition in the first place. And the situation is the same with every time, even the smallest. In any such time I think only the successive progression from one instant to the next, where through all the parts of time and their addition a determinate time magnitude is finally produced. [A162–63/B203]

The act of synthesis that overcomes sensibility’s innate ways-of-separation is *movement* in space and *progress* in time. Both are effects of my essential active nature, the ontological referent of the “I think.” Movement, intending the “next” position, is how I manifest my counter-force to reality in the service of spatial unification; progress, intending the “next” moment, is how I emulate passing. These acts are essentially additive. Extensive magnitude is the magnitude of summation: the parts are what is presented as real; they “make possible (and hence necessarily precede) the presentation of the whole.”

Axioms

Euclidean space is the object of the axioms and theorems of geometry. These are necessary truths about space. Their quality as spatial is contributed by the form of outer sense. The fact that it is the domain of invariant relations of magnitude (axioms of

geometry) is the result of the necessity flowing from the fact that the “I think” exercises its unitary actuality through line-drawing. Thus these axioms

express the conditions of sensible a priori intuition under which alone the schema of a pure concept of outer appearance can come about—e.g., the axioms that between two points only one straight line is possible; or that two straight lines enclose no space; etc. These are the axioms that, properly speaking, concern only magnitudes (*quanta*), as such. [A163/B204]

But there is also knowledge of *mere magnitude*—i.e., how long something is in space, how long something lasts in time, and how many somethings there are in space or over time. This is knowledge of *quantitas*, or *number*. There are no synthetic a priori axioms of arithmetic. True equations are not axioms, because they are *singular propositions*. In “ $7 + 5 = 12$,” I construct the same number *twice*, and this identity tells me nothing new. (On the other hand, it is also not analytic, because the way the two sides of the equation are “thought” does not yield identity.) The axioms of geometry are universal, because there is some leeway in how I make my constructions. “A triangle” can be any of an infinite variety of individuals that are *actually* different in intuition. “7” can never be different in intuition, because it counts whatever I take as a “homogeneous item” as a unit. I can count anything, and it is the pure act of counting, making distinct posits over the course of passing and then totalizing them, that constitutes the sense of the product entirely. But “a triangle has an interior angle sum of 180° ” is a universal judgment, and thus a theorem or an axiom.

Body—the hidden category

Space and time are generated by combining pure point-moments through line-drawing. Consequently, any empirical data filling these points are combined as well. In fact, the apprehension of appearances actually occurs as the apprehension of *pure* point-moments, which happen to contain real sensations. Thus

what geometry says about pure intuition holds incontestably for empirical intuition also. ... The synthesis of spaces and times, which are the essential form of all intuition, is what also makes possible the apprehension of appearance, hence makes possible any outer experience, and consequently also makes possible all cognition of the objects of this experience. [A165–66/B206]

Reality *for me*, that is, reality that I apprehend by actualizing my identity across space, is thus bound together into the same *necessary* unity that I myself have as subject. The result of this binding across space is not only magnitude, but also *real unity*. Point-data are not individuals lying next to one another; rather there is a *body*, reality-in-extension. Kant does not include *body* among the categories, but its membership follows from the reality of *substance*. *Substance*, as we will see, is reality that perdures through time. That is, reality *occupies* and *pervades* time. The reality here is borrowed from the fact of stimulation. My being-stimulated contains the quality of reality, because it is extra-volitional. Now, the content that is produced in me passes away as soon as it arises. But then it is replaced, and replaced constantly. This constancy of force, the force behind my being stimulated, is a feature of reality. I counter it by forcing a continual posit. This makes my reality manifest, in intuition, as a force: despite passing, it is constant. This I do by positing a point and moving it, where motion is the effect of my force that emulates passing. By doing this, I manifest my apperceptive identity *through* time. I am an identical reality: identical in consciousness and act, and real by being a potency for stimulation (I stimulate my own passivity when I draw a line in the imagination). Reality, then, is inseparable from the synthesis of magnitude. Time itself is magnitude, and time itself is *substance*.

But the same must hold true of the product of this synthesis in space. Space is a *presented* plurality-in-unity that makes the factually un-presented plurality of passing presentable. My reality spans *across* space with just as much continuity of reality as it does *through* time. This invisible but ontologically essential attribute of space is,

however, *body*. Thus the extensive magnitude produced by the subject-position through line-drawing (remember that Kant ties the judgment-forms of Quantity to the subject-position) also produces extended reality. So when *sensible* reality is given, its connection to other point-data is one of continuous existence. Matter cannot be unextended in space, just as it cannot be serial in time.

Magnitude (intensive): Anticipations of Perception

Between the given and its absence

Point-data are filled with empirical content. This, the content produced by my stimulation, is “the real of sensation” [A166/B207]. But I am also aware of a datum as a pure (empty) point-moment. I can imagine it “empty” of any content. The empty and the filled contents relate to each other as zero and some value *n*. This is an a priori insight, produced by the application of the judgment-forms of *affirmation* and *negation* to none other than the content of reality. But what results is another span of extensive magnitude. I am quantifying sense content by its “intensity.” Relating this value to zero can only be achieved by line-drawing and counting. This line is extensive magnitude, but I am interpreting it under the combined rule of *affirmation* and *negation*, or the *infinite* judgment-form. Now the line means something else.

- A Edition: “The *principle* that anticipates all perceptions, as such, reads thus: In all appearances sensation, as well as the *real* that corresponds to it in the object (*realitas phaenomenon*), has an *intensive magnitude*, i.e., a degree” [A166].
- B Edition: “Their principle is: *In all appearances the real that is an object of sensation has intensive magnitude*, i.e., a degree” [B207].

Interpreting the line non-extensively

We have seen how by drawing a line I produce the significance of space, time, *quantum* (continuous magnitude), and *quantitas* (discrete magnitude, or number). And due to the fact that my activity is the emulation of reality, and that my reality spans across and *inhabits* space and time through the act of drawing, objective reality also pervades space and time—as *body* and *substance*.

Space, time, *quantum*, *quantitas*, *body*, and *substance* are all magnitudes. But they are specifically magnitudes *of extension*. They are magnitudes that are presented in intuition as the extension of pure intuition, as formal intuition itself. The very images of magnitude *are* space and time. I extend from parts, from point-moments that I intend and thereby fill with my attention, and from this make a whole. I move and progress across what was formerly a plurality, and fill it with my reality. Extensive magnitude is a concept whose object *is* spatial and temporal extension.

Now Kant will use line-drawing once again to produce *quantitas*. But this time, the produced value will receive an additional interpretive intent. Instead of intending the magnitude as referring to *actually* (i.e., spatially) extended images of magnitude (sub-spaces and sub-times), I intend it as referring to a magnitude that is not presented sensibly in in space at all.⁶⁷ This is the magnitude of the *quality* of a point-datum's *content*. The object here is unextended in space and is momentary in time. It is the “intensity” of a point-moment. Hence, Kant notes, the force of reality is called a “moment,” since it is instantaneous [A169/B210]. The magnitude is in the content of the datum. With the principle of *limitation* (which Kant calls the principle of *reality*, a misnomer since it is

⁶⁷ For this reason *quality* can be schematized as an axis in analytic geometry that is *orthogonal* to the axis presenting actual spatial (temporal) extension.

not the given reality I think, but its alterity within the second-order quality under which I subsume it), Kant has reoriented the concept magnitude away from the forms of intuition.

Making the predicate a continuously changing value

The value of the above principle for mathematical physics lies in the fact that I can now put the content of a point-datum (its state-value, which is a property-magnitude in the continuum of quality, thought under the predicate-position) into a relation with its time-position (a magnitude in the continuum of substance, thought under the subject-position). The subject- and predicate-positions, in their transcendental function as rules of synthesis, both have schemata that are also *magnitudes*. Variation in state can now correspond to variation of position in the time-continuum. This is the condition of the possibility of mathematical laws of state-change. In other words, the important thing about quality as a value-continuum is the anticipation that *change* will instantiate infinitesimally, that is, *continually*. No new value can leave a gap of mediating difference:

every sensation is capable of diminution, so that it can decrease and thus gradually vanish. Hence between reality contained in appearance, on the one hand, and negation, on the other hand, there is a continuous coherence of many possible intermediate sensations, whose difference from one another is always smaller than the difference between the given sensation and zero, i.e., complete negation. In other words, the real contained in appearance has always a magnitude. [A168/B209–10]

Space, time, and quality are all continuous (or “flowing”) magnitudes. Continuity reigns over space, change in position, time, and change in time-position. And now it also reigns over every quality. A property *could be* any other on the second-order continuum of quality, and if it *does* change to a non-*P*, it will do so in a way that no gap of state-value is presented in the transition between moments. “Hence all appearances as such are continuous magnitudes—both in terms of their intuition, viz., as extensive magnitudes,

and in terms of their mere perception (sensation, and hence reality), viz., as intensive magnitudes” [A170/B212]. Change in quality occurs as movement in a continuum, the very means by which I time and space (and body and substance) are themselves constructed. We will now be able to interlink state as property or accident to substance or essence—the invisible reality flowing through time, making time into a real substrate. We will now be able to clock physical objects and relate their state-change to an internal law, “in” the substance. Things have properties that are determined by laws of change. There can now be an a priori science of change, because state is a quantity that is related to another quantity, and quantities can relate through arithmetic as ratios. Ratios, in turn, can hold between entire continuums by means of algebraic relations. The ratio “ $y = 2x$ ” is a relation between two continuums of real numbers. By taking one variable to be *independent* and the other *dependent*, I can determine the value in one variable from the value of the other. This “from” is the grammatical basis of the category of *cause*. Causality is the mathematical predetermination of state as a value of time. This is made possible by the judgment-form of limitation, which schematizes the range of the predicate as a continuum of value-differences, as in our previous *red* example. *Red* means not non-*red*, and both lie on a continuum of *one* way-of-difference, this being the second-order *quality* itself. *Red* is a range within *hue*, which also contains all of non-*red*. Making quality a continuum of real numbers lets me put it into relation with the continuum of real numbers resulting from totalizing time as a series of infinitesimals, or moments. For every moment in objective time, or substance, there is a state-value.

Were it not for the fact that the ratio or law that determines the time-series of objective quality-values depended on the contingency of real contents being given, “the proposition that all change (a thing's transition from one state to another) is likewise

continuous could be proved here easily and with mathematical self-evidence” [A171/B213].

The Analogies of Experience

- A Edition: “Their general *principle* is this: All appearances are, as regards their existence, subject a priori to rules governing the determination of their relation to one another in one time” [A176–77].
- B Edition: “Their principle is: *Experience is possible only through the presentation of a necessary connection of perceptions*” [B218].

We have already anticipated this principle in our comments on causality. What is interesting is that Kant has given a principle for the heading of Relation, meaning that all three of the categories listed there share this one principle in common. What is at stake is the difference between the *real* order of states in time and the *accidental and subjective* order in which I apprehend these states. The latter is determined by whim. I can visually scan a physical object in any spatial order: top-to-bottom, diagonally, left-to-right, etc. But this order of my apprehending is not (necessarily) reflected in reality: “in experience the relation within the manifold’s existence is to be presented not as the manifold is compiled in time, but as it objectively is in time” [B219]. There is a time in which I apprehend, and there is time *as it is*.

Permanence, succession, and simultaneity

Take the example of a given segment. I can trace over it starting at one end or the other. But by apprehending it either way, I put its constituent data into the temporal relation of *succession*. This is the “subjective time-order” of my act of apprehension. But, as we supposed, in the *objective* time-order, the data belong to a objectively spatial

segment, and stand in the temporal relation of *simultaneity*. Or take the example of a rock. The pixels constituting the body of the rock stand in the relation of *simultaneity*; but, again, I have no choice but to trace over them sequentially. While the contents of its pixels (the intrinsic properties of the rock) are contents that are *sequential* values of a quality, this is independent of the combination of these pixels in the objective time-order into the simultaneity of a body. But there is something else: the reality of the rock, its substance, is *permanent*. The reality that causes my continual stimulation relates to itself through time as *self-identical*. Thus the data constituting the rock have been combined in three different ways—into substance, into properties, and into a body. These relations—**permanence**, **succession**, and **simultaneity**—Kant calls the “three modes of time” [B219].

Kant assigns each mode to a different judgment-form—categorical, hypothetical, and disjunctive. The inclusion of the latter two judgment-forms in his Table of Judgments is, as I have suggested, one of the two mistakes in Kant’s selection of judgment-forms. This is because only atomic judgment is really necessary for apperception. We have already seen that the disjunctive and hypothetical judgment-forms are forms of inference that properly belong to reason, not the power of judgment. Moreover, Kant himself locates the rule for the determination of states in objective time in the copula, as we have seen. The second mistake, also previously mentioned, is the inclusion of the modal operators, under the heading of Modality. Kant’s objective modality is no different from logical modality, except for making the “time condition” necessary for the application of *necessity* and *possibility* explicit, i.e., that necessity entails *for all times* and possibility, *some times*.

The three modes of time are *permanence*, *succession*, and *simultaneity*. Hence there will be three rules governing all time relations of appearances, whereby

every appearance's existence can be determined in regard to the unity of all time; and these rules will precede experience and make it possible in the first place.
[A177/B219]

What is being determined is *reality*, something that has meaning for me only through positing in intuition. Connecting pixels as reality, then, can occur in three ways. The plurality of the real as permanent is an imaginary synthesis of reality as perduring, and belongs to *substance*. When this plurality is simultaneous, it must be spatial, and thus in a *body*. And when this plurality is taken in succession, I cognize a sequence of *properties*, ordered in objective time, by a rule—that is, state-value determined by time-value, or *causality*.

Relations of existence

The mathematical principles of Quantity and Quality are “constitutive” because they govern the mathematical synthesis that actually “assembles” objects of intuition (i.e., mathematical objects). The dynamical principles of Relation are “regulative” because they govern the “combination” that denotes the meaning of *existence*.

The mathematical principles of Quantity and Quality are constitutive because they determined facts that are actually present in intuition, i.e., in space. The figure and color of a body, for example, are facts falling under the principles of Quantity and Quality that are both presentable in space. The mathematical principles of *body* and *quality* allow me to produce, from a rule, an object of intuition—i.e., an imaginary spatial object. I can determine the length and figure of an intuition, and fill its second-order qualities to some determinate value.

The principles we are dealing with now, however, determine facts about how pixels relate to “existence” by being thought as interconnected according to the three modes of time (permanence, succession, and simultaneity). Kant calls this the thought of

the appearances' "*existence* and their *relation* to one another in regard to that existence" [A178/B220].

In the case of the mathematical categories of extensive and intensive magnitude, "the rule of the appearance's synthesis can also give this a priori intuition, i.e., can produce the appearance from this intuition, in the case of every empirical example that comes to hand" [A178/B220]. Mathematical synthesis yields determinate knowledge of particular objects (magnitudes) since knowing the rule of construction allows one to construct the object corresponding to that rule. And this synthesis has objective reality since appearances that fall under the categories of magnitude are themselves mathematically constructible. Kant calls the principles mathematical because "they justified applying mathematics to appearances, dealt with appearances in regard to their mere possibility; and they taught us how appearances could be produced, as regards both their intuition and the real in their perception, according to rules of mathematical synthesis" [A178/B221].

Kant distinguishes mathematical from dynamical analogy. In mathematics, analogies (or ratios) are "formulas asserting the equality of two relations of magnitudes, and are always *constitutive*; so that if three members of the proportion are given, the fourth is thereby also given, i.e., it can be constructed" [A179/B222]. Given the mathematical analogy "1:2 just as 5: n ," I know a priori that n must be "10." The rules of mathematical construction allow us to construct particular objects (magnitudes) according to fixed relations already known to hold among all magnitudes. But while an analogy of mathematics is constitutive in that it lets us posit the fourth member when three of a proportion (intensive or extensive) are given, an "analogy of experience" is only regulative: "Here, I can from three given members cognize, and give a priori, only the *relation* to a fourth, but not *this* fourth *member* itself" [A180/B222]. The fourth in its

particularity is not given, only a rule for “seeking” the fourth member is available. An analogy of experience is a rule guiding the unity of experience alone as thought through its corresponding schema. What Kant means by “only the *relation* to the fourth” will now be explored in our analysis of the particular subsections of the Analogies.

Substance: First Analogy

The category of substance as rule of intuition as such is “the concept of something that can exist as a subject but never as a mere predicate” [B149]. The schema of substance is that of “a substratum which ... endures while all else varies” [A144/B183]. Now, in the First Analogy, Kant announces the principle, or necessary truth, that holds of *objects* due to the schematism carried out by the subject-position:

- A Edition: “PRINCIPLE OF PERMANENCE—All appearances contain the permanent (i.e., *substance*) as the object itself, and the mutable as its mere determination, i.e., as a way in which the object exists” [A182].
- B Edition: “PRINCIPLE OF THE PERMANENCE OF SUBSTANCE—*In all variation by appearances substance is permanent, and its quantum in nature is neither increased nor decreased*” [B224].

“All appearances are in time; and solely in time, as substrate (viz., as permanent form of inner intuition), can either *simultaneity* or *succession* be presented” [A182/B224]. Time *itself* is what I cognize as permanent. But within time, point-data are interrelated with each other through two dimensions or media of separation—spatial and temporal. When I am aware that two data occur at the same (objective) time, then I have “determined” them according to the mode of *simultaneity*. When I am aware that they

occur at different (objective) times, I have determined them according to the mode of *succession*.

Kant's point is that I could only do this if spacetime itself were one, infinite, transparent, and invariant (permanent) block. The permanence of time as an all-containing substratum is what allows for relations of simultaneity:

In order to give, as corresponding to the concept of *substance*, something *permanent* in intuition (and thereby establish this concept's objective reality), we need an intuition *in space* (an intuition of matter); for space alone is determined as permanent, whereas time, and hence whatever is in inner sense, constantly flows. [B291]

The principle of permanence is needed in order to explain how it is possible for me to distinguish between subjective and objective time-orders. Making this distinction is a puzzle under Kant's theory of figurative synthesis since time and space are *both* products of apprehension, *which is successive*. I produce a span of unified space by moving a point—a time-taking activity. I produce a span of time in the same way, while attending to passing rather than to changing position. Given this fact, how am I able to distinguish between what is really simultaneous (even though I perceive it successively) from what is really successive, i.e., what is in fact a change in states of what is permanent?

Permanence as identical act

For Kant, permanence denotes the invariant block of spacetime, the coordinate system that I necessarily refer to when I am aware that two point-moments “exist” in the same moment or successively in time.

When I consider the relation between two point-moments, whether spatial or temporal, I must draw a line between them in order to apprehend their separation as a relation. By doing this, I relate them “in” a unity. This unity is originally the unity of my

act of drawing—one (identical) act spans its way as a power of positing through many point-moments, verifying their possession by one acting subject. Kant is now saying that permanence is the objectified result of this identity-across. I am an identical agent of positing across space in a time-irrelevant way, and I am identical through time in a space-irrelevant way. The original act is the hybrid one of line-drawing that creates change through motion. Despite this, I am able to separate the two kinds of relation in experience.

Take the example of a table. I must apprehend the passing pixels into a body—point-data that are objectively simultaneous. But apprehending takes time. If I am tracing the table from bottom-left to top-right, then the points in the top-right will be “later” than those in the bottom-left. Yet I am aware that this succession of points is false, that it only belongs to the *subjective* time-order of my act of apprehending. What allows me to be aware of this is the structural invariance of spacetime. I can move a point, in my imagination, between two points in space and know that this is only a subjective exercise superimposed on points that are objectively simultaneous only if *nothing changes while I move*. For “time itself” to change during the act of line-drawing would entail nothing other than the disruption of the awareness of my own self-identity.

If objective time changed while spanned through it, I would not be spanning through objective time. Instead, objective time would become part of the successive activity of subjective time. Thus: “If we wished to attribute to time itself a succession or sequentiality, then we would have to think yet another time wherein this succession would be possible” [A183/B226]. The permanence Kant talks about is in fact none other than the stability of spatialized time. When I move about in spatialized time, every time-position is *determinate*. If things moved around, then I would not be aware of the identity

of my act, since the very coordinate system through which I was moving would be in a constant flux.

Causality: Second Analogy

The principle of causality is the culmination of the entire positive program of the First *Critique*. In the preface to the *Prolegomena* Kant identifies the destruction of metaphysics, of necessary truths about reality, with Hume's attack on causality: "Since the Essays of *Locke and Leibniz*, or rather since the rise of metaphysics as far as the history of it reaches, no event has occurred that could have been more decisive with respect to the fate of this science than the attack made upon it by *David Hume*" [P 4:257]. This attack began with a simple analysis of causality, but it ended with the impossibility of metaphysics in general: "*Hume* started mainly from a single but important concept in metaphysics, namely, that of the *connection of cause and effect* (and also its derivative concepts, of force and action, etc.)." Hume proved that this connection cannot have a "rational" basis—i.e., it cannot rest on a containment relation between universals in analytic judgment. From this Hume concluded that "there is no metaphysics at all, and cannot be any" [P 4:258].

The connection that Hume could not establish in analytic judgment was "that something could be so constituted that, if it is posited, something else necessarily must thereby also be posited; for that is what the concept of cause says" [P 4:257]. Hume found that the notion that one appearance contains a "power" that necessitates the nature of its successor cannot be established empirically. Sensibility establishes point-data, but not powers. Neither can this power be established by pure rational inference. What seems like a real connection is actually a fiction:

All events seem entirely loose and separate. One event follows another; but we never can observe any tie between them. They seem conjoined, but never

connected. And as we can have no idea of any thing which never appeared to our outward sense or inward sentiment, the necessary conclusion seems to be that we have no idea of connexion or power at all, and that these words are absolutely, without any meaning, when employed either in philosophical reasonings or common life. (Hume, *Enquiry*, Section 7)

Representationalism to the rescue

All facts about noumenal reality are irrelevant, so it does not matter if intellectual intuition of their nature is not possible, and it does not matter that our pure inferences by themselves cannot establish a necessity that is, for us, spatiotemporal. Space and time are internal ways of pluralization—stimulation is a priori spatialized, and the passing of stimulation is also noted as an a priori way-of-difference. These are internal because they are the sensible forms of epistemic consciousness itself.

This consciousness is unitary, and this unity becomes active when I reflect on my ontological nature as agent, or spontaneity. When spontaneity becomes self-propelled, I can make intuitions in imagination. This is the act that effects the assertive intention of “is P.” I make images that look like given intuitions. When I know the rule of image-making, I own the universal (the name of this rule) and can assert its homogeneity with a real intuition. This act is also unitary, and this makes my ways-of-separation subject to a demand for combination, or synthesis. I am present as agent wherever I can posit, and I unify this by moving the point I posit, not by positing again. The movement injects the unity of the agent across space and through time, bringing them to the unity of a magnitude, since combination that *retains* is additive.

Now, these forms of my sensible reception are also the forms of my sensible contents, and thus of objects. So the things that we should really be investigating, Kant says, are *these* objects—the only ones I can have, all of which are therefore subject to my *media* of separation, my *rules* of combination (“This (S) is P”), and my *active* essence as

unitary agent. Appearances are carried along with the ways-of-separation that I combine into “This (S) is P.”

The subject-position contains a rule of intellectual synthesis, i.e., logical combination, and this innate form now looks for, and so constructs in lieu of its absence, numerical quantity—the spatiotemporal analog of logical quantity. *Some* and *all* can apply to the sense world only numerically: *all* of these chairs are green only after I count all the chairs under “these” and then count all of “these” green chairs. Only numerical relations can deliver the possible application of the logical quantifier to cognition. Reality *extended* in space is unified under the rule *body*, or real spatial magnitude. But I can also span my identity across the real-through-time, and make time as the image of magnitude. Doing this is also carried out by logical quantification that, Kant says, is contained as a function of the subject-position.

The property is the non-substance that I subsume under the predicate-position. And its meaning is the rule that I use to produce its image. This rule is the command to make a continuum of values, one of which is the actually instantiated property.

Now the property is a real number. I can quantify any property, and Kant says that I *always* do so whenever I consider a second-order property that subsumes an instance. I see a red intuition, and I know that it could be ... paler, darker, richer, more orange, more violet, etc., and all these are magnitudes. This a priori relation, which is the very meaning of the rule that produces this *limitation* on a continuum of variability, that a particular property has to the continuum of differences to which it belongs, is the sensible effect coerced by the intellectual freedom to *affirm* and *deny*. I know what truth is by knowing how it can fail—how I can make an image that is *not* contained in the logical complex thought in the subject-position. But also how to make it falsely *within the same kind*. I know, a priori, failure within a kind. I know this because I make the image from a rule

that spans a continuum of alterity. When I know that something is *red* I know a meaning that is affirmed out of a continuum of differences which give *red* meaning. I know that this is not orange, yellow, green, etc. This is the *infinite* judgment-form, and it is how the predicate coerces a continuum of differential property values within a kind-delimiting quality.

The predicate is a state-value continuum just as the subject is either a spatial continuum (if thinking the object's body in abstraction from its situation in the flow of passing) or the temporal continuum itself, permanence of the real through time, or substance, which is just the spatial nature of the presentation of time, since space is the very basis of the sense of *objective permanence* (the subjective basis being the identity of my act, the "original" basis of permanence and all other pluralities-in-unity).

These are the figurative syntheses enforced by the subject and predicate-positions. Now Kant treats our a priori awareness that the time-order of the state-values in objective time (i.e., in the physical world that I make through figurative synthesis) is *not accidental*. Every point-moment is filled with the content-value that in fact fills it by necessity. And this necessity is *necessary position in time*. Every point-datum is somewhere in time, and I believe a priori that the state-values of these data are *determined as a function of time*. A time-position "calls for" a certain value, and what determines this value is nothing other than that position, which is a moment, or point, in the line of magnitude by which time itself is originally produced. This is the principle the follows from the objective reality of the condition-conditioned relation. Kant calls it a principle of production, of producing a specific state-value at every time, so that the succession of state-value in objective time is *lawful*:

- A Edition: “PRINCIPLE OF PRODUCTION—Everything that occurs (i.e., starts to be) presupposes something that it succeeds *according to a rule*” [A189].
- B Edition: “PRINCIPLE OF TEMPORAL SUCCESSION ACCORDING TO THE LAW OF CAUSALITY—*All changes occur according to the law of the connection of cause and effect*” [B232].

Kant says that this determining of state-values in a law-determined sequence through time is the effect of the *hypothetical* judgment-form. This, as I have mentioned, cannot be correct. First, I can intend the concept of cause–effect simply by uttering “causes” in an atomic (categorical) judgment. Second, my judgments of objective fact routinely do not involve the hypothetical form. I make judgments of the structure “This (S) is P” all the time, without reliance on material implication, although I can invoke this.

The fact that causality is really the power of the copula follows from simply paying attention to what Kant has done so far, and to how he has defined the schema, or rule of figurative synthesis, that we must use to make the sense of *causality*: “The schema of the cause and of the causality of a thing as such is the real upon which, whenever it is posited, something else also follows” [A144/B183]. The real here is the state-value of a substance. In objective time, all state-values are determined by their position. This is the mode of *succession*. Succession occurs lawfully, and this just means that every state-value is where it is in the sequence of objective time by necessity. This has a consequence in our perception of the object’s existence-in-time. And this is the perception of states following each other by necessity: when *a* is the value given at an instant, then *b* must be next. This necessary following is not produced by the preceding state, but this is the force that time-determination has on the synthesis “in the object.”

Kant shows how this transformation in meaning takes place with an argument. If reality gives me the series *a-then-b*, then since it is my imagination that does all sensible combining, this series could have been reversed. The argument then proceeds to say that since I *did*, in fact, experience *a* and then *b*, this must be accounted for. Whence an awareness of *necessitated* or *real* succession, i.e., one that is irreversible *for a reason*? This kind of teleological pull towards irreversibility could only be the coercive effect of a rule of understanding. These are the judgment-forms at work in truth-claiming, which motivates me to invoke the rule that produces a matching image of an intuition—that is, to assert “S is P.” This rule says that *P* is *a* and then later *b* by necessity; that is, that the order *a-then-b* is *fixed*. This fixity of time-positions is what is inherited by time from its spatialization. This fixed order puts the states *a* and *b* themselves into a relation with each other, so that the collection of state-values contained in datum *a* is perceived as *itself* causing the collection of values to follow, in *b*.

But the real basis of a necessary time-sequence is nothing other than the rule stating that every state-value is determined by its position in time. Knowing the *objective* order of time is knowing how state-values are laid-out in time. This is a relation between values and a *line*. Every point in the line has the particular state-value that it has due to the magnitude of time that contains it. At $t = 1$, *P* is *a*; at $t = 2$, *P* is *b*. The values *a* and *b* are *real numbers* (continuous magnitudes) and their having fixed positions in time just means that these values have been determined by 1 and 2. Time itself, the permanent framework of invariant moment-relations, is the line I produce under the subject-position, when I intend an object for the unity of the subject-position *over time*. This is *substance*, one of the referents (along with *body*) of the subject-position. But the referent of *P* is itself also a continuum of real numbers. Causality is the mathematical determination of the value of one variable (intending state-value) by the value of another (intending time

as a line). The former is the *condition* of the latter: this is the definition of the subject–predicate relation in general logic. This is the real conditional judgment-form at work in our cognition of causality. The *categorical* judgment-form contains the essence that Kant ascribes to the *hypothetical* judgment-form. And it effects this, finally, as the algebraic relation between an *independent* and a *dependent* variable, through the copula, which is the algebraic-functional assignment operator “ \rightarrow .”

This is how I am able to distinguish between the objective time-order and the subjective order of my apprehension. My apprehension is what unifies both spatial and temporal separation, so every datum for me lies in a time-series. Nonetheless, I am able to bifurcate my experience into the order that is really “out there,” and the order that is the artifact of my perceptual route through it. This must be accounted for, and it can only be explained as something that I impose in my act of molding data into conformity with my ability to judge “This (S) is P” while aiming my intentionality towards passing pixels. I am the source of all unity, because all combination is post-reception by sensibility. This awareness of the succession that is *really there* (which relies on my awareness of when data are related by simultaneity, i.e., spatially) must then be the effect of a rule. And since my stance is epistemic, it must be a rule of understanding, i.e., a judgment-form.

Kant explains this awareness of necessary succession as the effect of a rule on the otherwise free power of imagination. I combine things so that *a*-then-*b*, and I am aware that this is fixed. But my imagination alone would not do this. Linking-in-time alone links without a set order:

In the imagination itself, however, the sequence is not at all determined as regards order (i.e., as to what must precede and what must follow), and the series of the presentations following one another can be taken as proceeding backward just as well as forward. But if this synthesis is a synthesis of apprehension (of the manifold of a given appearance), then the order is determined in the object, or—to speak more accurately—there is in this apprehension an order of successive

synthesis that determines an object; and according to this order something must necessarily precede, and when this something is posited then the other event must necessarily follow. [A201/B246]

Two additional arguments

from substance as action—Kant once again reinforces my thesis that it is not the hypothetical but the categorical form that is really at work in our cognition of causality. He does so by arguing that cause, as action, is an empirical criterion for substances: “causality leads to the concept of action; action leads to the concept of force and thereby to the concept of substance” [A204/B249]. What makes a substance sub-standing as a unity is its action as a rule that forces state-values. Substance is active: it is the rule that produces the value of a datum. This power, Kant says, is an “empirical criterion of a substance insofar as it seems to manifest itself not through the permanence of appearance but better and more easily through action” [A204/B249].

The power of cause is nothing other than substance *as permanence*, that is, as fixed time-order. This is the pure fixed-time order, but its effect in sensibility is the determination of state-values in this time-order: “the ultimate subject of the mutable is the *permanent* as the substratum of everything that varies, i.e., substance” [A205/B250]. Kant goes on: “For according to the principle of causality actions are always the first basis of all variation by appearances; hence actions cannot reside in a subject that itself varies, since otherwise other actions and another subject determining that variation would be required.” This explicitly locates causality in the subject-position as a fixed order that determines “all variation by appearances,” i.e., the order of state-values. In fact, Kant says, we can infer the permanence of substance *from* causality: “that the first subject of the causality of all arising and passing away cannot itself arise and pass away (in the realm of appearances) is a safe inference that issues in empirical necessity and

permanence in existence, and hence in the concept of a substance as appearance” [A205–6/B251].

from continuity—Finally, Kant says, continuity is essential to causality: “all change is possible only through a continuous action of the causality” [A208/B254]. As time progresses, state-value changes continuously, so that each moment presents a value that is infinitesimally different. This establishes precisely the relation that I have argued is really at work in Kant’s theory of ostensive judgment. This is the relation between permanence, or the line of time-magnitude, to quality-continuum, of the line of possible state-values. The two vary together, according to a law, and according to an order that is time-forward:

Now every change has a cause that manifests its causality in the entire time wherein the change takes place. Hence this cause produces its change not suddenly (i.e., all at once, or in one instant), but in a time; so that, as the time increases from its initial instant (*a*) up to its completion (in *b*), the reality’s magnitude ($b - a$) is also produced through all the smaller degrees contained between the first degree and the last. Hence all change is possible only through a continuous action of the causality; this action, insofar as it is uniform, is called a moment. Change does not consist of these moments, but is produced by them as their effect. [A208/B253–54]

Causality is a determination relation between time-magnitude and “the reality’s magnitude.” The progression of state-value is *smooth*—there are no leaps. In mathematical terms, the curve that is produced by graphing this relation is differentiable. Kant’s principle of the continuity of state-change, the basis of the calculus of infinitesimals through which mathematical physics constructs the very laws of nature, is embedded directly in the schematism of the structure of judgment, “S is P.”

Kant has argued that the grammar of natural language used in cognition of reality as “S is P” is really, that is, *schematized* through the pure forms of sensibility, through acts of line-drawing that construct time and quality-continuum, and then relates these as *S*

and P through an “objective” copula, through “the little relational word *is* in judgments” [B141]. This relation is one that determines one magnitude (state-value) from another (time -position). This is nothing other than the relation of the independent time variable to the dependent state-value variable through an algebraic function: $t \rightarrow f(t)$. When “S is P” faces the manifold of spacetime, it carries out imaginary syntheses that construct magnitudes and relates them determinately, according to a mathematical law that can be discovered. Every law of change is a mathematical function of time, and this is the Kantian theory of natural grammar.

Conclusion

THE STAGES OF SENSIBLE KNOWING

The unified physical object of understanding is the first combination effected by the effort to understand. Here, the understanding synthesizes merely to *present* an object—an object that is not yet known. But for an object to be *known*, it must be an object that a *judgment* can assert in truth. It must be analyzed into a collection of marks, these marks must be recombined into a logical object, and I must then consciously extract and re-apply one of these marks as a predicate in judgment. In this way, I consciously reconstruct the object as a combination of universals in the structure of judgment. This creates a truth-claim. To then *justify* this claim (and produce knowledge) I need to verify that what I assert in judgment corresponds to, or is “homogeneous with,” the object that I am claiming to re-present. To do this, I must first render the emulation that I assert in judgment as a *particular* object, since I can only compare one particular object to another. This capacity is called the **power of judgment**. The ability to “subsume” an intuition is really the ability to produce a likeness of it through imagination, which is how my nature as active power determines *and thereby epistemically accesses* my passive power of intuition. This is done by **schematizing** the judgment’s component universals. When I see that the schematized (imaginary) object corresponds with the given (real) one, truth is justified and knowledge is attained.

So understanding carries out various tasks: it combines point-moments into a physical object, notices marks that it can abstract as universals, combines these universals into a logical complex, and re-combines them *again* consciously and under will by actively asserting this combination in judgment as truth. Finally, the understanding schematizes this assertion as an image that can be compared to the original object. These

acts are distinct and have different names but are all carried out by the same faculty of understanding:

1. The combination of point-moments into a physical object is variously called “transcendental synthesis of reproduction”, “synthesis of productive imagination, and “**figurative synthesis.**”
2. The ability to compare physical objects and abstract universals as analytic predicates through the process of reflection is called “the logical act of comparison, reflection, and abstraction,” or more simply the **process of reflection.**
3. The act of constructing a logical complex of universals is the act of **logical combination.**
4. The act of *asserting* this combination as an analytic truth, by extracting a universal from the complex and then re-attaching it via predication, is the act of **analytic judgment.**
5. But an analytic judgment has sense only by being about possible intuitions. The act of following the procedural rules for image-making named by the subject and predicate concepts is called “**schematism,**” and the rules “**schemata.**”
6. And, finally, the ability to compare my ability to make any image with the particular now before me in intuition is called the “**power of judgment.**” These are all different functions of the same understanding.

THE TRANSCENDENTAL DEDUCTION

The Transcendental Analytic is about steps (1) and (5) in the above list. In the Transcendental Deduction, Kant explains how the understanding blindly combines passing pixel arrays into physical objects that conform to “S is P.” In the Schematism, he explains how I carry out this same act *consciously*, as an emulation in the imagination.

Truth

Automatic synthesis is carried out in the service of knowledge. I am aware *epistemically* by asserting “S is P,” which makes a truth-claim. This is an act: I create “S is P” by my intention. But it also refers to an object in intuition, real or imaginary. In order for “S is P” to be a *possible* truth, the object must itself have the structure “S is P.” This can occur in two ways: by making the object in form and content myself as a logical combination of universals (potential predicates), or by making an object that is a collection of various ways-of-combination in the domain of sensibility, i.e., the ways that are picked out by the subject- and predicate-positions and then combined in the copula.

The logical object is a collection of universals in logical combination. Their relation to each other is what I mean by “is” in non-ostensive judgment, i.e., judgment whose terms are universals, e.g., “All men are mortal.”

The sensible object is a collection of ways of spatiotemporal combination. These ways are *necessary*. The Transcendental Deduction explains why they are necessary. The answer is that, first, judgment itself is necessary for awareness. Second, the subject of knowledge is unitary, but sensibility delivers a plurality. This entails a necessary combination—the pluralities that I (a unity) am aware of must be (for me) pluralities-in-unity. These pluralities are ways-of-separation, also a priori. These are spatial separation, temporal separation, and logical separation. I am aware of these as I face sensibility, so I must have combined them; and I must have combined them into unities that conform to the subject- and predicate-positions and their relation through the copula. But this time, I am not relating two *universals*, I am relating a extensive magnitude to an intensive magnitude. The subject-position a priori picks out and subsumes a *body* that is a *substance*. That is, it subsumes spatial magnitude and temporal magnitude. The predicate-position picks out a *property* and a *quality*. A *property* is an insubstantial (momentary)

instantiation of a kind, realized in a body. A *quality* is a continuum of different property-instances that can be realized and which fall under the same second-order type. Finally, the copula places the continuum of state-values into relation with the continuum of time. It says that the state-value which is instantiated at every moment of objective time is *determined* by law. This is the origin and meaning of *causality*.

Two conditions of awareness

What are the conditions of sensible awareness? There are two. First, awareness has the form “S is P.” Second, awareness is unitary while its object is a plurality-in-unity.

Sensible awareness has the structure “This (S) is P,” where *P* is a concept, or rule of image-production. To know an intuition is to know it as *P*, and to know that something is *P* is to know the rule, called “P,” that can produce a certain class of imaginary instances, including the one before me. This awareness is **apperception**. To be an apperceptive subject is simply to be the subject of an object *that is understood*. Being aware has certain conditions, one of which is the ability to imagine what I am aware of. I see blue, but I am aware of seeing blue only if I can produce a matching image. The ability to produce an image is for Kant the very possession of meaning. Being aware means at least this much: I am aware of *that* only if I am aware of *what* “that” is. This is the same as asserting “is P” of *that*.

Unitary awareness of plurality entails combination

The second piece of Kant’s argument flows from the fact that the “I think” occurs, and that it is aware of *all* of the passing plurality of data. This means, first, that it is aware of each, by being able to produce an pure image of the formal simple of sensibility, the point-moment. It does this by positing point. Second, it means that it has combined this plurality (and therefore its posited points). To be aware of the spatiotemporal plurality

means being aware of it as a unity, and thus *as* spatiotemporal. These spatial and temporal ways-of-separation are combined into formal space and time—pure images of magnitude.

THE SCHEMATISM

Meaning is the ability to produce instances

Now space and time *mean* something to me. Meaning is the ability to produce and image for a kind. In this case, the object (and corresponding) image is unique—it is singular, it is non-empirical, and it is a product of additive combination. When I make space and time, I make empty continuants that can *receive* contents because these are open frameworks that *locate* data but do not provide them. Nonetheless, these unique images, because they *do* meaning something to me, must be producible by following a procedural rule.

Combinations that mean something to me are ones that I can emulate: they have sense, and so must be producible like images must be producible from the rule associated with any universal. They are combinations that *originate* from a combination that is spontaneous and unconscious, but they are also ones that I can carry out intentionally—as they must be in order to have sense and significance.

But *space* and *time* are not images of the familiar empirical kind, such as images of *green*, *triangle*, and *dog*. They are extraordinary images—pure ones, ways-of-combination across and through the ways-of-separation in which my passive subjectivity splays-out the continual stimulation of outer sense. In order to make them consciously, I carry out the all-important act of line-drawing.

Basic line-drawing

Line-drawing is the basic act of figurative synthesis and the realization of the necessary unity of apperception. Remember that the self that I am aware of is the subject of sensible knowledge. It is also, ontologically, the power of the subject to produce the image that realizes its awareness. Any datum must be makeable if I am aware of it. The element of appearance is a point-datum, and its forms is the pure (empty) point-moment. To be a knowing power of image-making at a point is simply to fill it *in any way*. I must be able to fill any point that is under my unity simply in order to verify that “this,” the simple locus of my attention in intuition, is *mine*. And so I draw a line in order to verify my presence as a knower that knows space as a “this.” The line is the image of my image-making authority over the pure form of outer sense. I posit a point and “move” it. Doing so maintains its (my) identity, thereby “positing” my necessary apperceptive identity as an image, which I can take as a spatial *continuant* or a temporal *continuant* depending on how I interpret my act. The act of positing the moving point is an act of power—I have affected my own outer sense and made a weak intuition in my imagination. This is how I know *with consciousness*: by positing an image from a rule that subsumes “this” under “P.” The result of this conscious production of space and time through line-drawing is that “space” and “time” now *mean* something to me. This, as we have seen, is identical to my being able to make them consciously.

Interpreted line-drawing

Now this combination obviously cannot be carried out by the forms that are being combined. We already know that it must be carried out by my agency of image-making following a rule. A rule named by an empirical concept is optional—it might or might not be predicated of an object, depending on whether the rule produces a matching image. But the rule that makes space and time out of plurality and passing is a necessary rule; I

cannot *not* apply it. But the only rules that are necessary would be ones that permit “S is P.”

But this rule is non-optional, not one that I have learned from carrying out the process of reflection and can choose to withhold. It is an automatic rule, one that is carried out unconsciously but necessarily. Space and time are meaningful because they are pure images that I make, and they are necessary images because I make them by combining innate media according to a judgment-form, and judgment is necessary for awareness.

Space and time are meaningful because I can make them. To make space or time, I consciously draw a line while attending to some feature of my act, and this attention is ruled by the judgment-form that automatically subsumes it. When I draw a line as space or time, Kant says, I attend to the fact that the unity in drawing is itself *additive*, and also that the end-product of this act is a continuous whole magnitude that I am able to part and re-combine. These are kinds of awareness: awareness of *unity*, *plurality*, and *totality*. They are how I relate a priori to something like magnitude in general logic—I can think the subject-concept wholly or partially subordinated under the predicate-concept. And, indeed, when I apply the whole/part relation to sensibility, I do so over *numerical* quantities, or *totalities* of the pure homogeneous in intuition—the mere forms that I have spanned, combined, and *added* into extension by means of line-drawing. This is how space and time are constructed under a rule, and why they are the very images of magnitude. My ability to produce images of space and time, which is my possession of their meaning or semantic sense, is the effect of line-drawing as an additive act of magnitude production, which I then a priori separate and re-combine into a countable totality. And this, Kant says, is the effect of the subject-position acting as a rule of

figurative synthesis. When I carry it out consciously, it is called a transcendental schema, or “transcendental time determination.”

Power of act as emulation of reality

Spanning space is an act of self-power. When I posit an image, I inject my potency into sensibility by affecting my own intuition. But affecting outer sense is the hallmark of reality, the only difference being that the latter is extra-volitional. Thus the act of positing as such emulates reality. Now, when I exercise my power not discretely but continually, by line-drawing, in order to consciously combine the forms of space and time, I inject my reality in extension. As a result, I think of the real itself as extended. This is a *pure* extension of reality—a pure continuant. When I draw a line to produce a spatial continuant, I produce **body**, which is automatically subsumed under the subject-position. But my reality also spans through time as a temporal continuant. This I construct by drawing while attending to the identity of the moving point. This produces the sense of **substance**, which is also subsumed under the subject-position. This is reality that transcends the arising and passing of empirical contents. It is meaningful because I produce its “image” in the imaginary activity of moving an identical point.

What about the content of sensation? This is subsumed by the predicate-position. This content is always passing because it occurs in inner sense. This can be emulated: my inner sense is always passing as I draw. Drawing can be appropriated to present passing just as it presents space and time. And this is what Kant says. Change is no exception to Kant’s principle of meaning—if it is meaningful, I must be able to make an “image” for it. Reason, Kant says, cannot make change “understandable to itself without intuition. And this intuition is that of the motion of a point in space; solely the point’s existence in

different locations (as a succession of opposite determinations) is what first makes change intuitive” [B292].

By drawing while attending to passing, the semantic sense of *change* arises. The object of ostensive judgment is the substantial body, automatically subsumed under the subject-position that guides its synthesis. But what varies in the object is automatically subsumed under the predicate-position, and so Kant concludes that the rule that lets me appropriate line-drawing to schematize *change* is the logical act of predication.

The predicates of an object subject refer to empirical contents that are thought of as **properties**, even in mathematical judgments, since the figure must be present in intuition by means of color-boundaries. Multiplicity of contents is inherent to the function of the predicate. A logical subject always has multiple predicates, and in sensation this multiplicity is over-time. But the predicate contains this variable content within a kind of unity, and this is the unity of the universal itself—the unity of kind. A concept is defined by Kant as what can serve in the predicate-position, and universals are abstracted in this position; it is their birthplace. The universal or kind is a unity that ranges over many particular images. These images must have something in common, but they must also differ in some way. They differ by qualitative difference, and again as a magnitude. It is a fact that any predicate can be quantified. This is a way-of-combination that is meaningful for me, so it is something I have already made and it is something that I can make consciously as an “image.” Kant says that this image is that of a *value continuum*.

I produce images of a kind by following a rule. Kant shows that this rule is one that arranges these images in a continuum of qualitative difference, or **quality**. This rule limits my free acts of image-production so that the images I produce (1) all lie within a certain limited range of difference, and (2) all differ from each other by magnitude. Any

instance of red can differ *more* or *less* from some arbitrary norm. This is a magnitude. Magnitude is always constructed through line-drawing, since the material image of is spatialized time. Once again, I draw a line with special attention paid to some feature of the act. I arrange qualities in a continuum of difference that is itself isomorphic with the real number continuum. I have constructed an emulation of how rule-following occurs when I apply a predicate to sensibility: contents come together into kinds by means of differing from each other by *degree*. This is how I appropriate line-drawing in the service of conjuring an image for what the predicate-position can refer to when I face the passing plurality of point-data. The judgment-form that lets me do this is *affirmation/denial*. I affirm a predicate when its instance is really given. But this affirmation depends on negation—I know *red* only if I know what *red* is not. But this negation is limited to a second-order type, *non-red*, which is a continuum limited to difference in color, not difference in type (such as size or texture). Reality is produced by *affirmation*, meaning depends on *negation* (other instances under the universal), but the rule of the kind is a limitation, which Kant attributes to the function of the “non-” operator.

Finally, there is the fact that I can distinguish between objective time-order and subjective-time order. What is the difference? Only that, in objective time, contents occur when and where they do according to law, that is, regardless what the state of the subject may be. Cognizing the “image” of objective time, Kant says, is the function of the copula, and not the hypothetical judgment-form. The relation of state to time is the meaning of **causality**. [B141–42]

Under representationalism, the intersubjective world is the world made by innate media of sensible reception that are apprehended under innate rules of judgment, i.e., of knowing that “This (S) is P.” By unifying the media themselves through acts of line-drawing, I make unitary space and time. By unifying the passing point-moments of real

empirical content into conformity with the structure of judgment, I make extended **bodies** that are perduring **substances**, whose **properties** are state-values of a higher-order **quality**-continuum. These terms have meaning only if I can produce their instances in the imagination. To do this I use the functions of unity in judgment as rules of “time determination.” But Kant has already that that time is originally produced through line-drawing. To determine time can only be to think my act of drawing (combining, synthesis) in some specific way. These ways are, on one hand, grammatical or syntactic. But if we take Kant’s theory of schematism as line-drawing seriously, these ways are also algebraic. The subject and predicate relate as time-variable and state-variable, and they do so such that the former *determines* the latter. This is the schema of **causality**. Thus Kant’s entire metaphysics of nature can be seen as something that really does emanate from the systematic unity of ostensive judgment. All of his principles of physics are mathematical because judgment itself is covertly mathematical.

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